DSC291: Machine Learning with Few Labels

Deep Generative Models / Generative Adversarial Learning

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Lecture 11, May 6, 2025

UC San Diego
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Outline

- Deep Generative Models
  - Generative adversarial learning

- Paper presentation:
  - Devanshi Garg, Shrenik Jain: “RHO-1: Not All Tokens Are What You Need”

Generative modeling

- In generative modeling, we'd like to train a network that models a distribution, such as a distribution over images.
- One way to judge the quality of the model is to sample from it.
- This field has seen rapid progress:

2009

2015

2018

Courtesy: Grosse CSC321 Lecture 19

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- This field has seen rapid progress:

Figure courtesy: Ian Goodfellow

Generative modeling

Midjourney, 2025

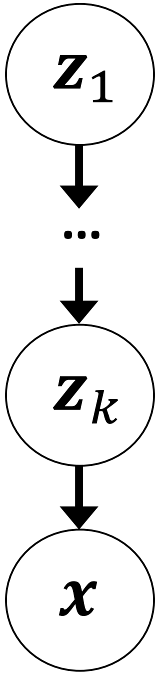
Generative modeling

- In ge… …bution,
  such…
- One…
- This…

Google Veo2, 12/2024

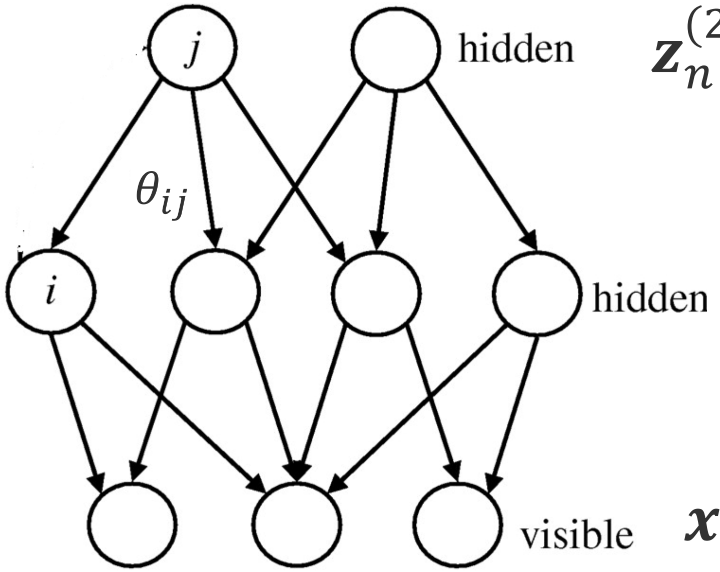
Deep generative models

- Define probabilistic distributions over a set of variables
- "Deep" means multiple layers of hidden variables!



Early forms of deep generative models

- Hierarchical Bayesian models
  - Sigmoid brief nets [Neal 1992]



$$\boldsymbol{z}_n^{(2)} = \{0,1\}^J$$

$$\boldsymbol{z}_n^{(1)} = \{0,1\}^I$$

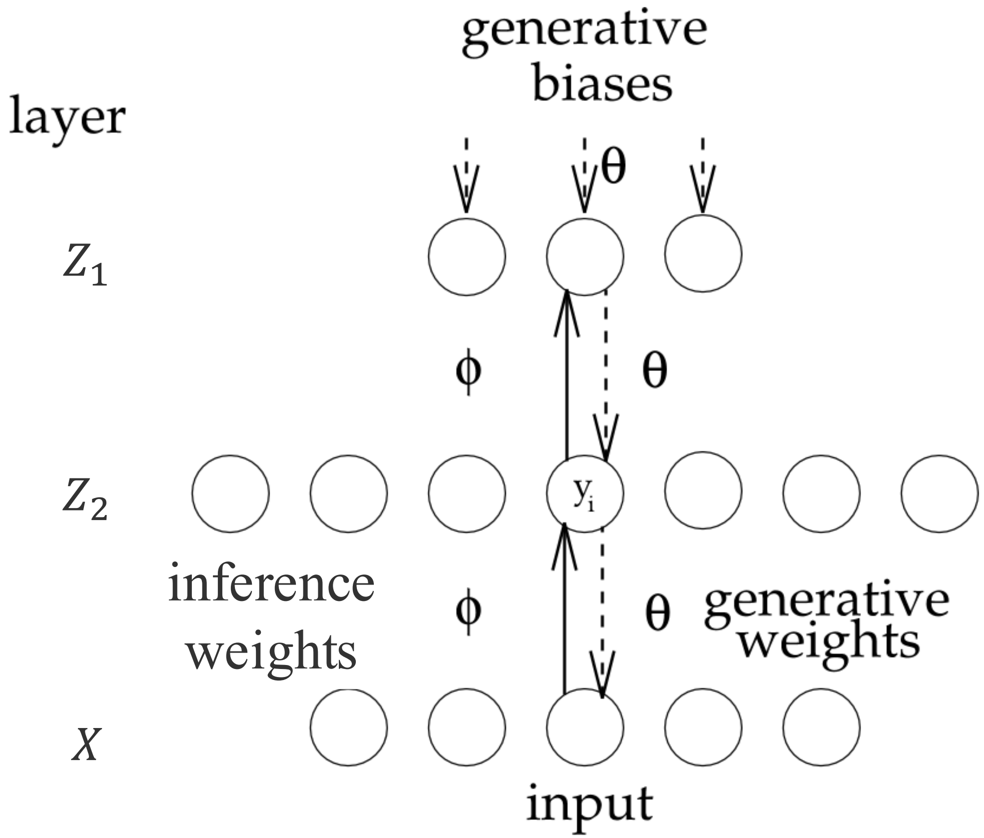
$$\boldsymbol{x}_n = \{0,1\}^K$$

$$p\left(x_{kn} = 1 \middle| \boldsymbol{\theta}_k, \boldsymbol{z}_n^{(1)}\right) = \sigma\left(\boldsymbol{\theta}_k^T \boldsymbol{z}_n^{(1)}\right)$$

$$p\left(z_{in}^{(1)} = 1 \middle| \boldsymbol{\theta}_i, \boldsymbol{z}_n^{(2)}\right) = \sigma\left(\boldsymbol{\theta}_i^T \boldsymbol{z}_n^{(2)}\right)$$

Early forms of deep generative models

- Hierarchical Bayesian models
  - Sigmoid brief nets [Neal 1992]
- Neural network models
  - Helmholtz machines [Dayan et al.,1995]



[Dayan et al. 1995]

Early forms of deep generative models

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- Neural network models
  - Helmholtz machines [Dayan et al.,1995]
  - Predictability minimization [Schmidhuber 1995]

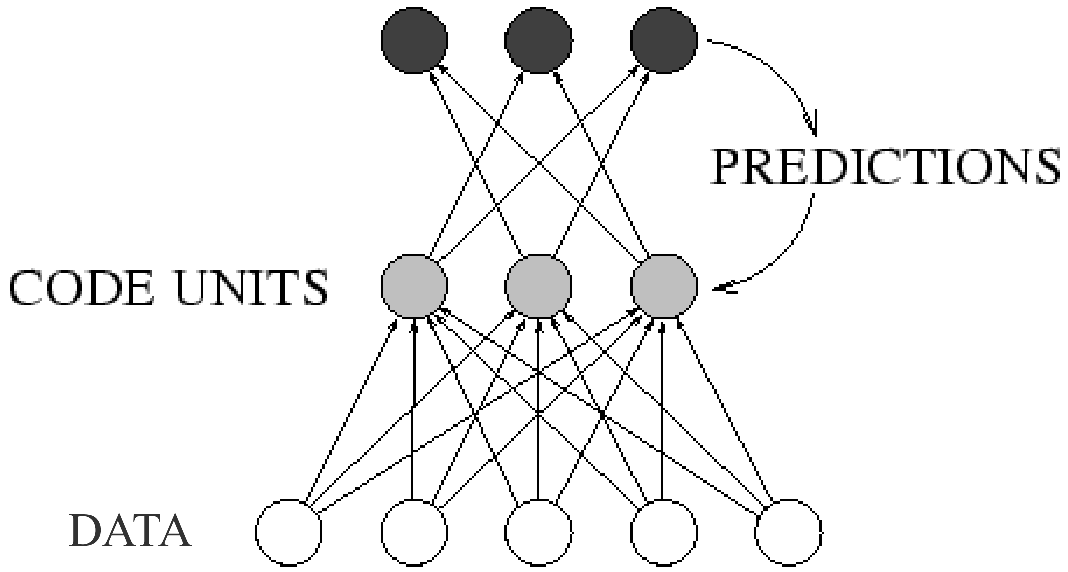
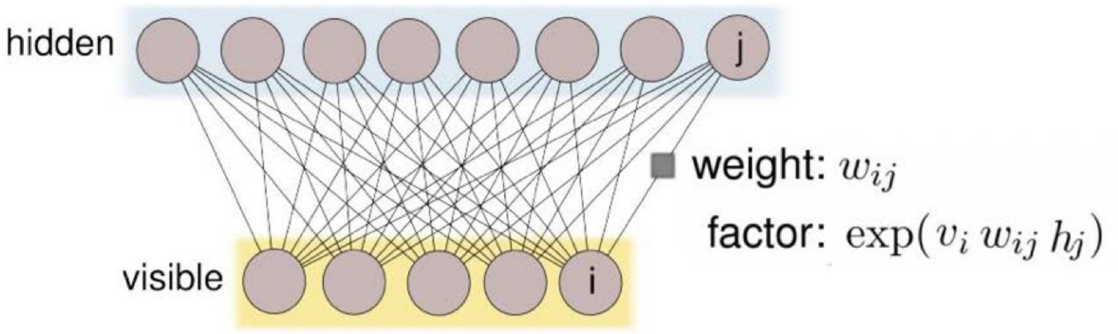


Figure courtesy: Schmidhuber 1996

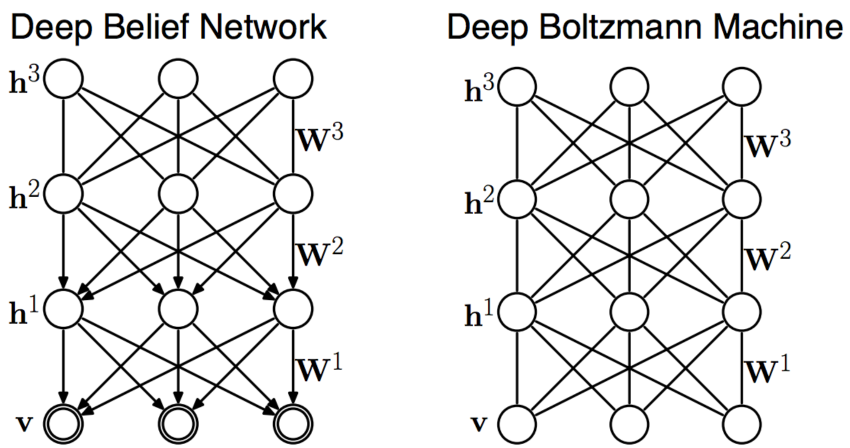
Resurgence of deep generative models

- Restricted Boltzmann machines (RBMs) [Smolensky, 1986]
  - Building blocks of deep probabilistic models



Resurgence of deep generative models

- Restricted Boltzmann machines (RBMs) [Smolensky, 1986]
  - Building blocks of deep probabilistic models
- Deep belief networks (DBNs) [Hinton et al., 2006]
  - Hybrid graphical model
  - Inference in DBNs is problematic due to explaining away
- Deep Boltzmann Machines (DBMs) [Salakhutdinov & Hinton, 2009]
  - Undirected model



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- Variational autoencoders (VAEs) [Kingma & Welling, 2014]

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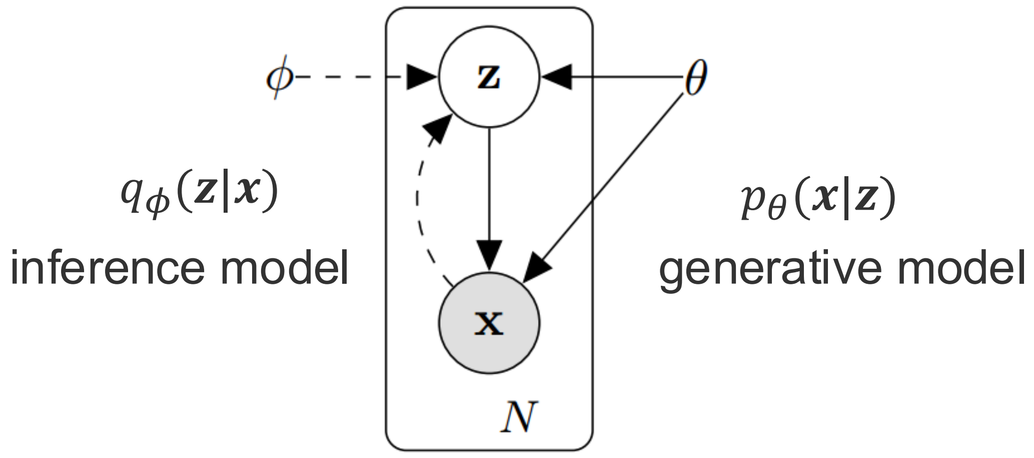

Figure courtesy: Kingma & Welling, 2014

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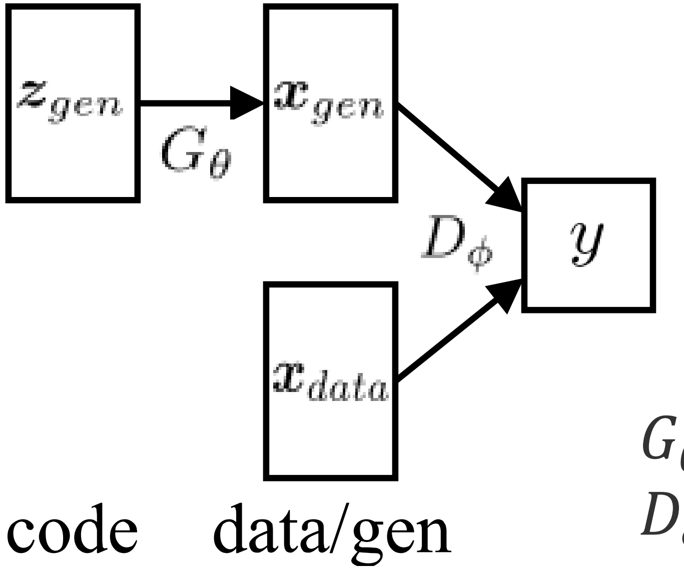

G_θ: generative model ?

D_ϕ: discriminator

Resurgence of deep generative models

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- Generative moment matching networks (GMMNs) [Li et al., 2015; Dziugaite et al., 2015]

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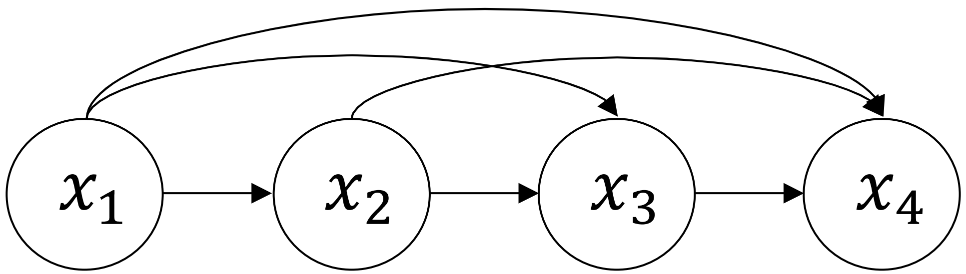
- Autoregressive neural networks



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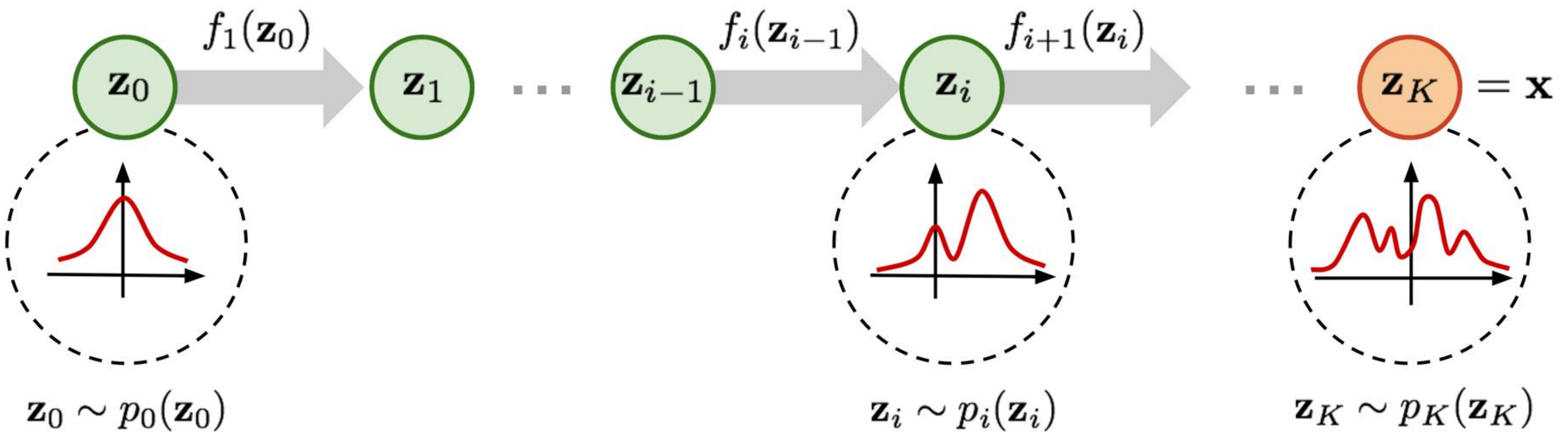
- Autoregressive neural networks
- Reversible architectures (flow models)



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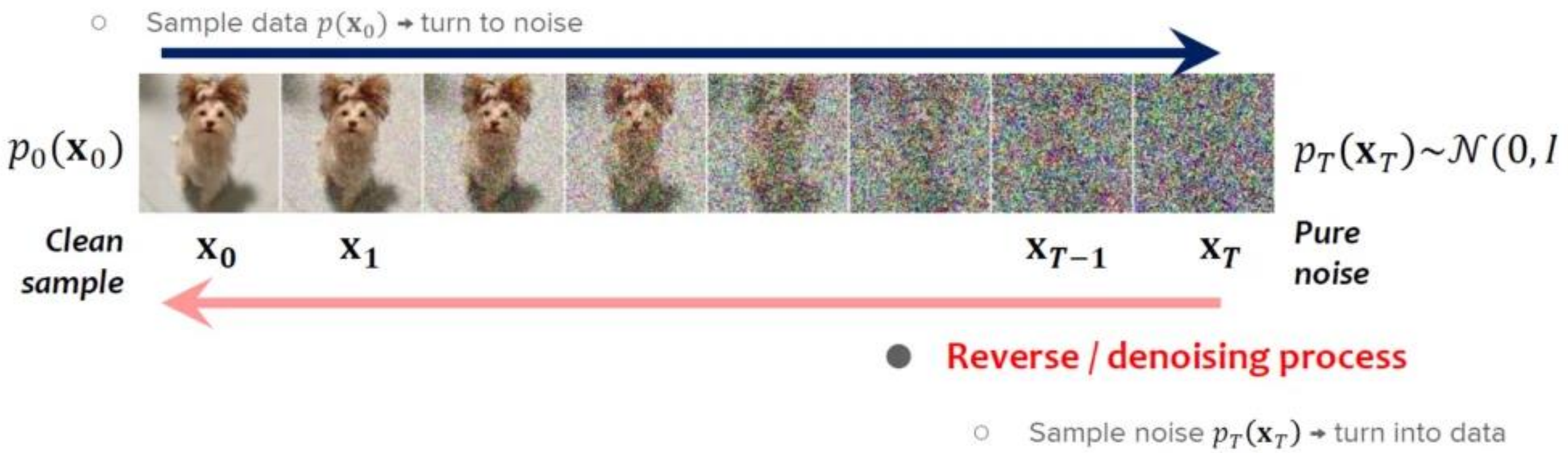
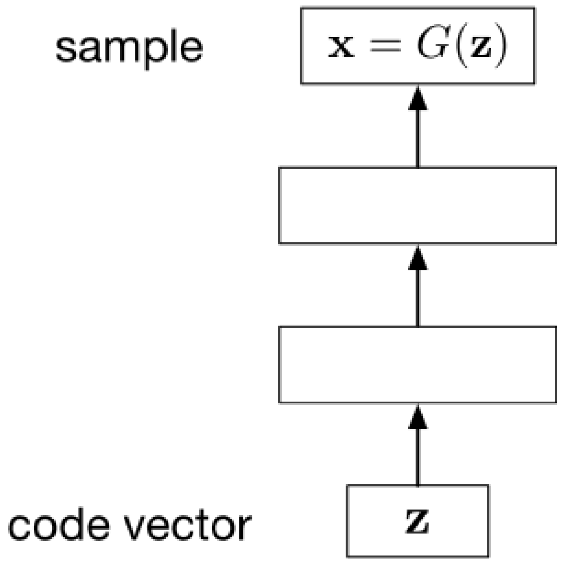
- Autoregressive neural networks
- Reversible architectures (flow models)
- Diffusion models



Generative Adversarial Networks

Implicit Generative Models

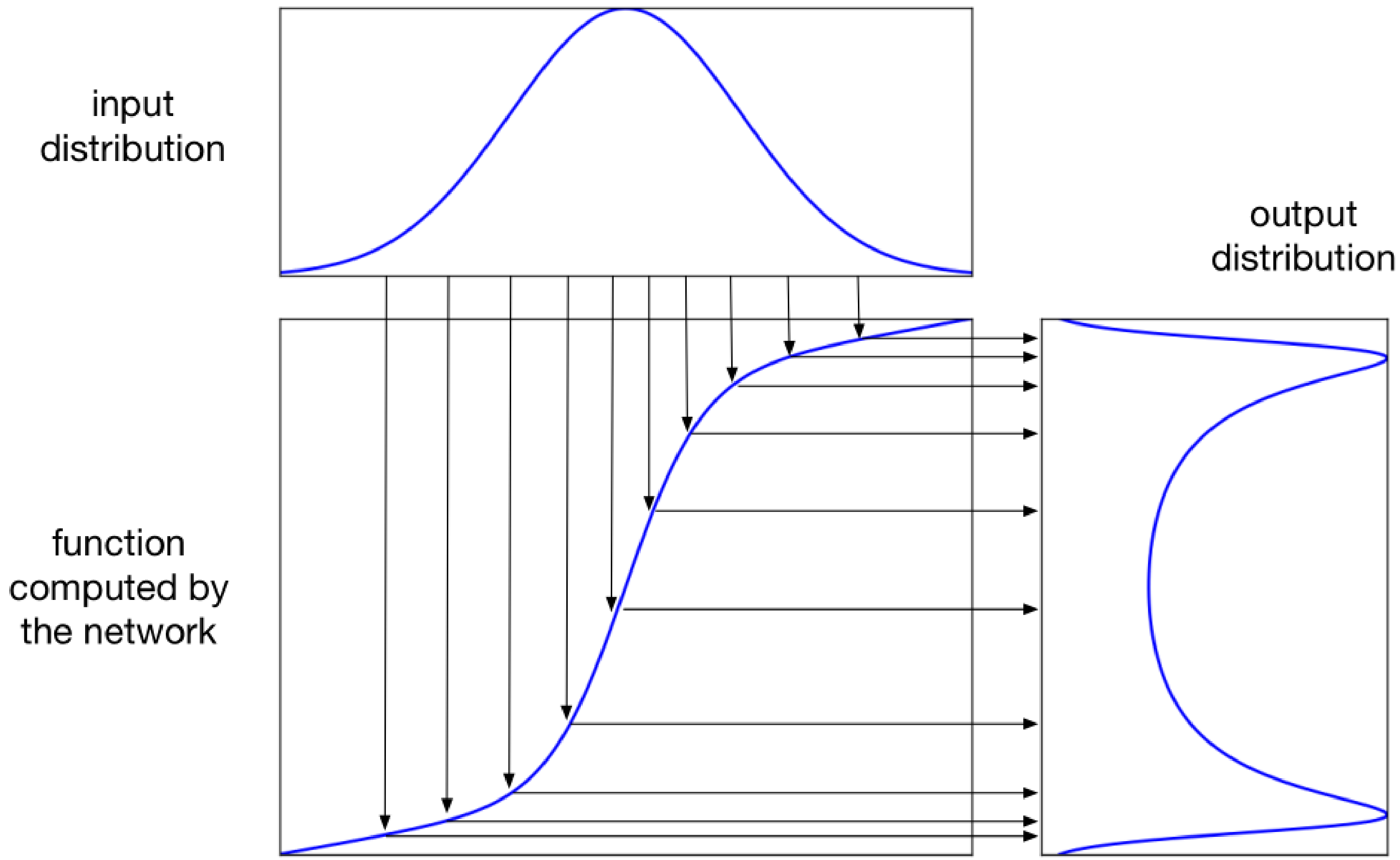
- Implicit generative models implicitly define a probability distribution
- Start by sampling the code vector **z** from a fixed, simple distribution (e.g. spherical Gaussian)
- The generator network computes a differentiable function G mapping **z** to an **x** in data space



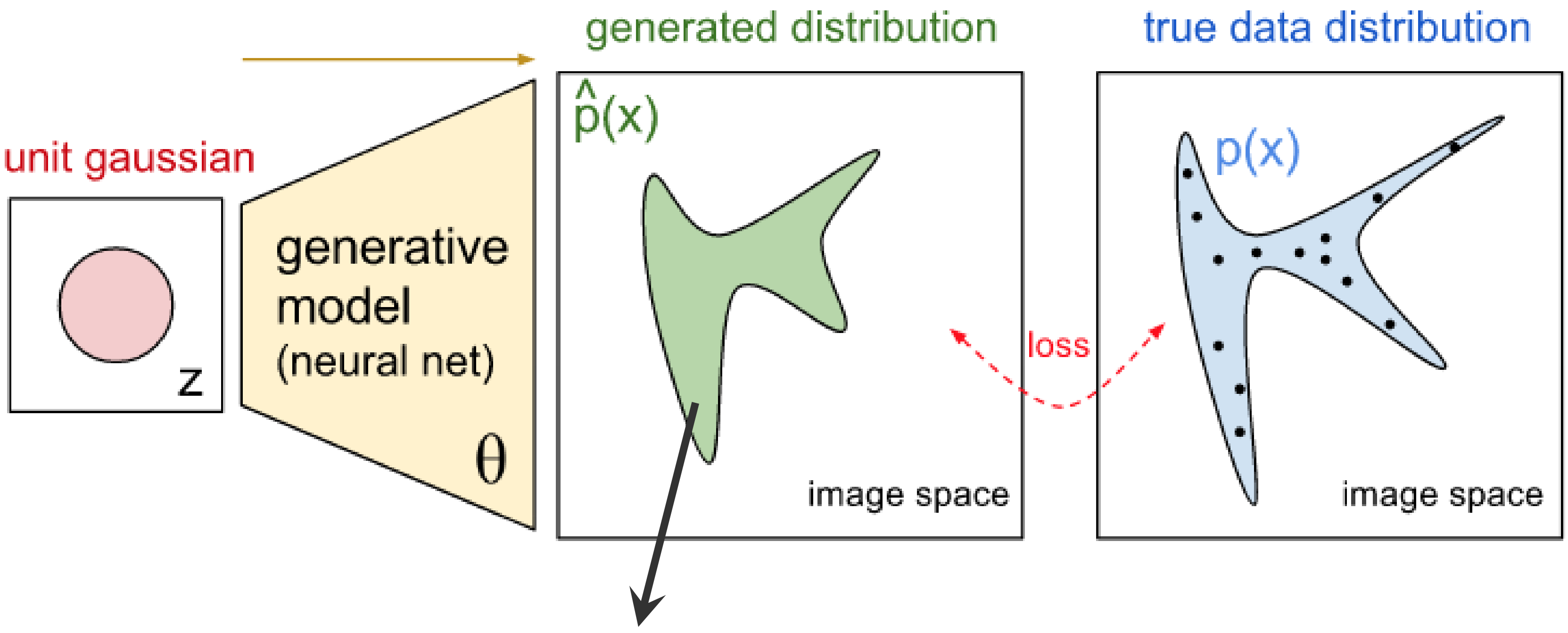
- a stochastic process to simulate data $\boldsymbol{x}$
- Intractable to evaluate likelihood

Implicit Generative Models

A 1-dimensional example:



Implicit Generative Models



https://blog.openai.com/generative-models/

Courtesy: Grosse CSC321 Lecture 19

Implicit Generative Models

- The advantage of implicit generative models: if you have some criterion for evaluating the quality of samples, then you can compute its gradient with respect to the network parameters, and update the network's parameters to make the sample a little better
- The idea behind **Generative Adversarial Networks (GANs)**: train two different networks
  - The generator network tries to produce realistic-looking samples
  - The discriminator network tries to figure out whether an image came from the training set or the generator network
- The generator network tries to fool the discriminator network

Generative Adversarial Nets (GANs)

- Generative model $\boldsymbol{x} = G_\theta(\boldsymbol{z}), \quad \boldsymbol{z} \sim p(\boldsymbol{z})$
  - Maps noise variable $\boldsymbol{z}$ to data space $\boldsymbol{x}$
  - Defines an implicit distribution over $\boldsymbol{x}$: $p_{g_\theta}(\boldsymbol{x})$
- Discriminator $D_\phi(\boldsymbol{x})$
  - Output the probability that $\boldsymbol{x}$ came from the data rather than the generator

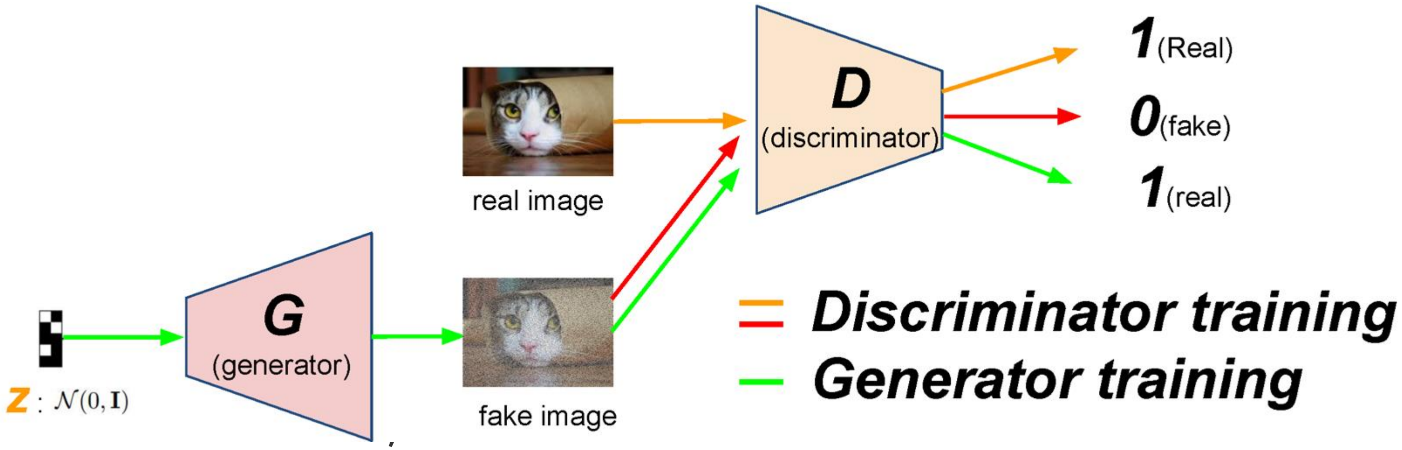


Figure courtesy: Kim

Generative Adversarial Nets (GANs)

- Learning
  - A minimax game between the generator and the discriminator
  - Train D to maximize the probability of assigning the correct label to both training examples and generated samples
  - Train G to fool the discriminator

$$\max_D \mathcal{L}_D = \mathbb{E}_{\boldsymbol{x}\sim p_{data}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x}\sim G(\boldsymbol{z}),\boldsymbol{z}\sim p(\boldsymbol{z})}[\log(1 - D(\boldsymbol{x}))]$$
$$\min_G \mathcal{L}_G = \mathbb{E}_{\boldsymbol{x}\sim G(\boldsymbol{z}),\boldsymbol{z}\sim p(\boldsymbol{z})}[\log(1 - D(\boldsymbol{x}))].$$

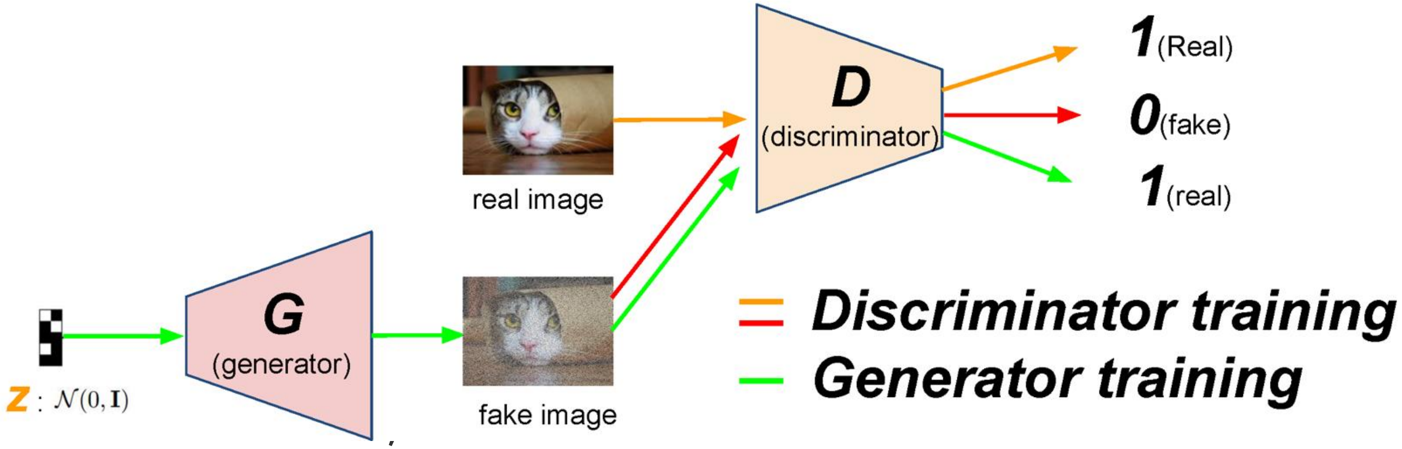
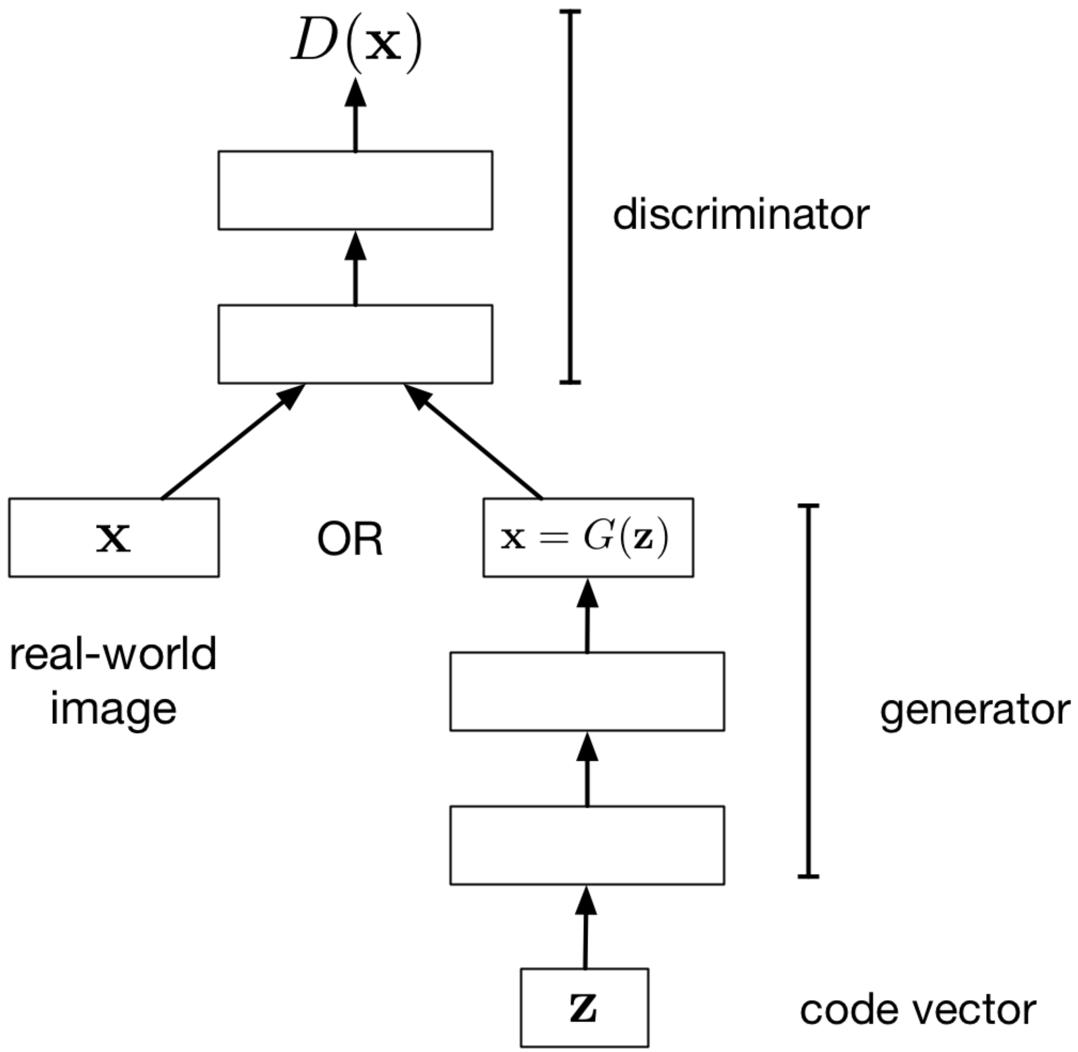


Figure courtesy: Kim

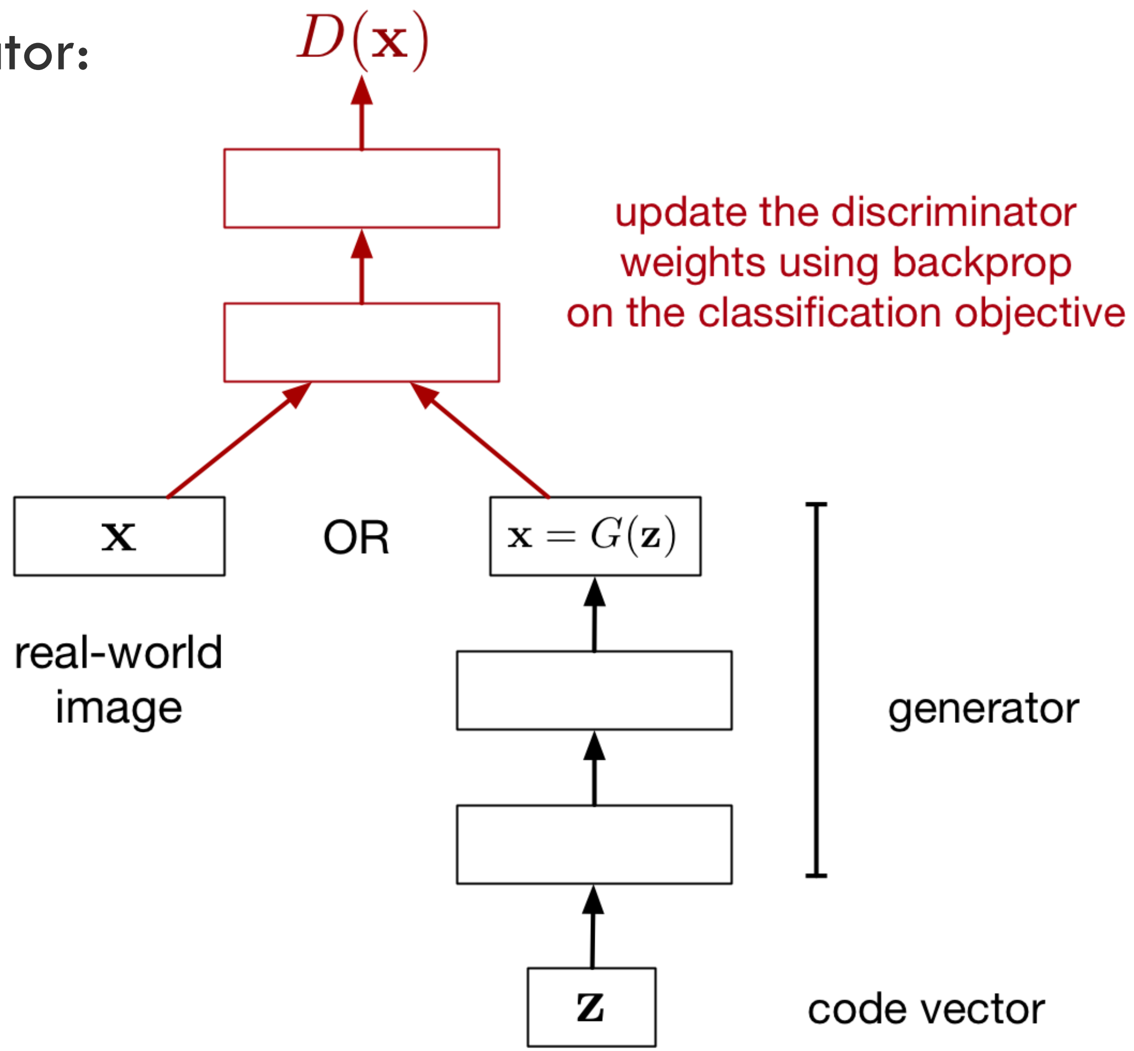
Generative Adversarial Nets (GANs)



Courtesy: Grosse CSC321 Lecture 19

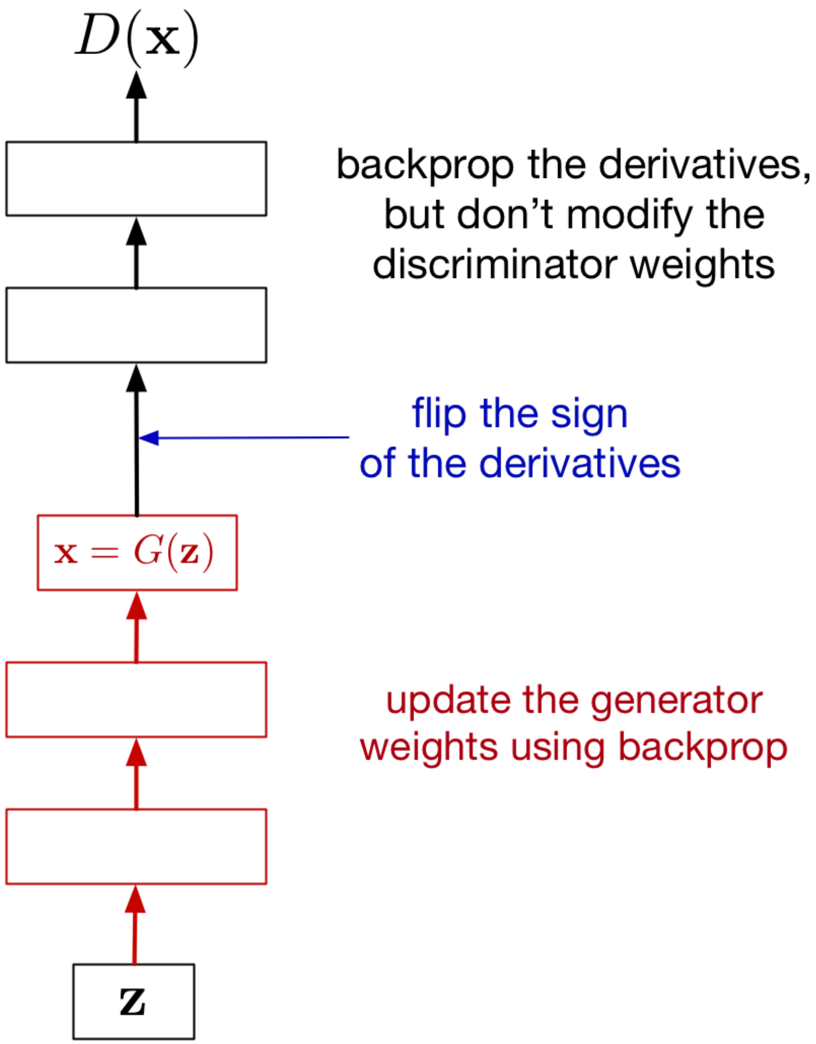
Generative Adversarial Nets (GANs)

Updating the discriminator:



Generative Adversarial Nets (GANs)

Updating the generator:



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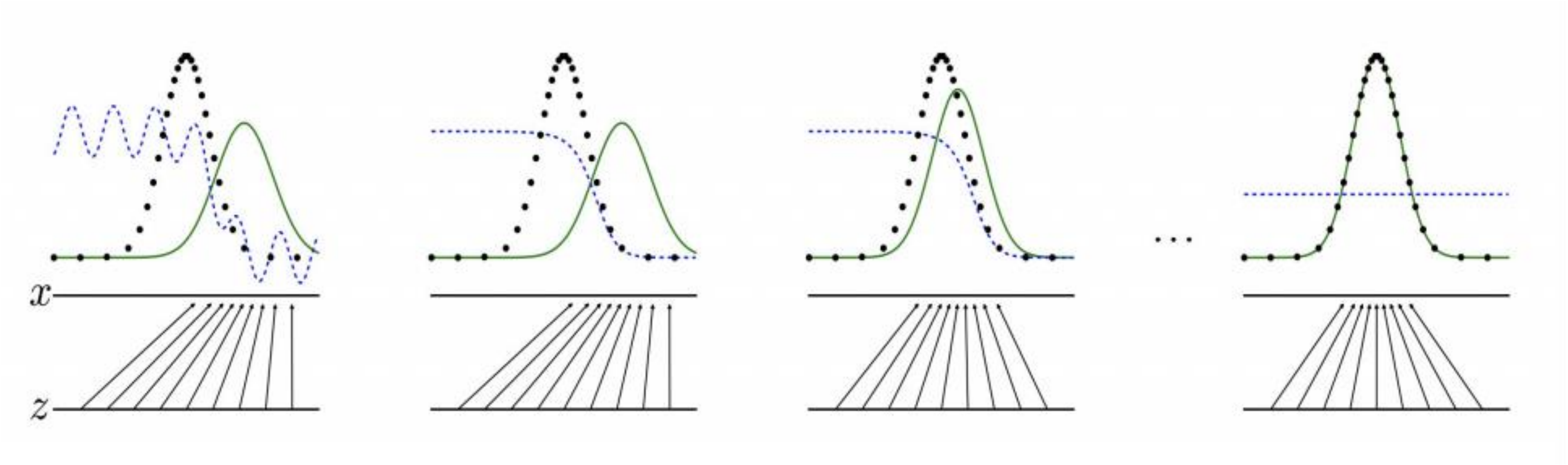
Generative Adversarial Nets (GANs)

Alternating training of the generator and discriminator:



Courtesy: Grosse CSC321 Lecture 19

Optimality of GANs

- Objectives:

$$\max_D \mathcal{L}_D = \mathbb{E}_{\boldsymbol{x} \sim p_{data}(\boldsymbol{x})} [\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x} \sim G(\boldsymbol{z}), \boldsymbol{z} \sim p(\boldsymbol{z})} [\log(1 - D(\boldsymbol{x}))]$$
$$\min_G \mathcal{L}_G = \mathbb{E}_{\boldsymbol{x} \sim G(\boldsymbol{z}), \boldsymbol{z} \sim p(\boldsymbol{z})} [\log(1 - D(\boldsymbol{x}))] .$$

- Global optimality: $p_g = p_{data}$
- Proof:

Optimality of GANs

Proposition 1. *For G fixed, the optimal discriminator D is*

$$D_G^*(\boldsymbol{x}) = \frac{p_{data}(\boldsymbol{x})}{p_{data}(\boldsymbol{x}) + p_g(\boldsymbol{x})} \tag{2}$$

[Goodfellow et al., 2014]

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Proof. The training criterion for the discriminator D, given any generator G, is to maximize the quantity $V(G, D)$

$$\begin{aligned} V(G, D) &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x}))dx + \int_z p_{\boldsymbol{z}}(\boldsymbol{z}) \log(1 - D(g(\boldsymbol{z})))dz \\ &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x})) + p_g(\boldsymbol{x}) \log(1 - D(\boldsymbol{x}))dx \end{aligned} \tag{3}$$

For any $(a, b) \in \mathbb{R}^2 \setminus \{0, 0\}$, the function $y \to a\log(y) + b\log(1 - y)$ achieves its maximum in $[0, 1]$ at $\frac{a}{a+b}$.

[Goodfellow et al., 2014]

Optimality of GANs

- The minimax game can now be reformulated as

$$
\begin{aligned}
C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}}[\log(1 - D_G^*(G(\boldsymbol{z})))] \\
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Theorem 1. *The global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_g = p_{data}$. At that point, $C(G)$ achieves the value $-\log 4$.*

[Goodfellow et al., 2014]

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$$
\begin{aligned}
C(G) &= -\log(4) + KL\left(p_{\text{data}} \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right) + KL\left(p_g \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right) \\
&= -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right)
\end{aligned}
$$

Jensen-Shannon Divergence

[Goodfellow et al., 2014]

Question: in practice, we're unlikely to get the optimal D^*. In this case, what is the minimax game truly optimizing?

Optimality of GANs

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Jensen-Shannon Divergence

[Goodfellow et al., 2014]

Wasserstein GAN (WGAN)

- If our data are on a low-dimensional manifold of a high dimensional space, the model's manifold and the true data manifold can have a negligible intersection in practice

[Arjovsky et al., 2017] Slide adapted from bhiksha

Wasserstein GAN (WGAN)

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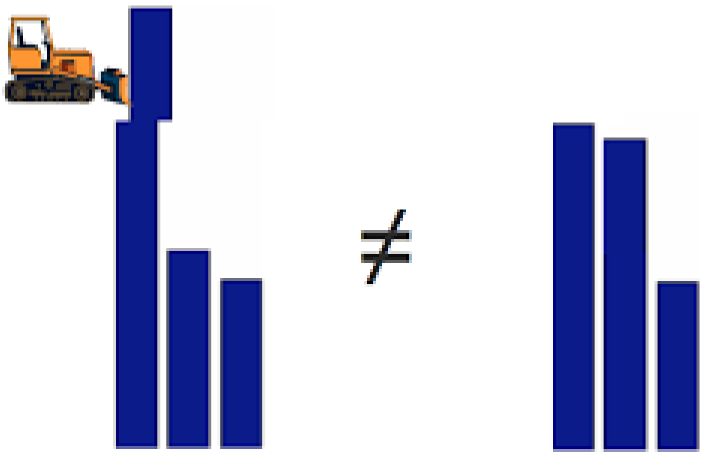
Wasserstein GAN (WGAN)

- If our data are on a low-dimensional manifold of a high dimensional space, the model's manifold and the true data manifold can have a negligible intersection in practice
- The loss function and gradients may not be continuous and well behaved
- The Wasserstein Distance is well defined
  - Earth Mover's Distance
  - Minimum transportation cost for making one pile of dirt in the shape of one probability distribution to the shape of the other distribution



[Arjovsky et al., 2017] Slide adapted from bhiksha

Wasserstein GAN (WGAN)

- Objective

$$W\left(p_{data}, p_g\right) = \frac{1}{K} \sup_{||D||_L \leq K} \mathrm{E}_{x\sim p_{data}}[D(x)] - \mathrm{E}_{x\sim p_g}[D(x)]$$

- $||D||_L \leq K$: K- Lipschitz continuous
- Use gradient-clipping to ensure D has the Lipschitz continuity

Progressive GAN

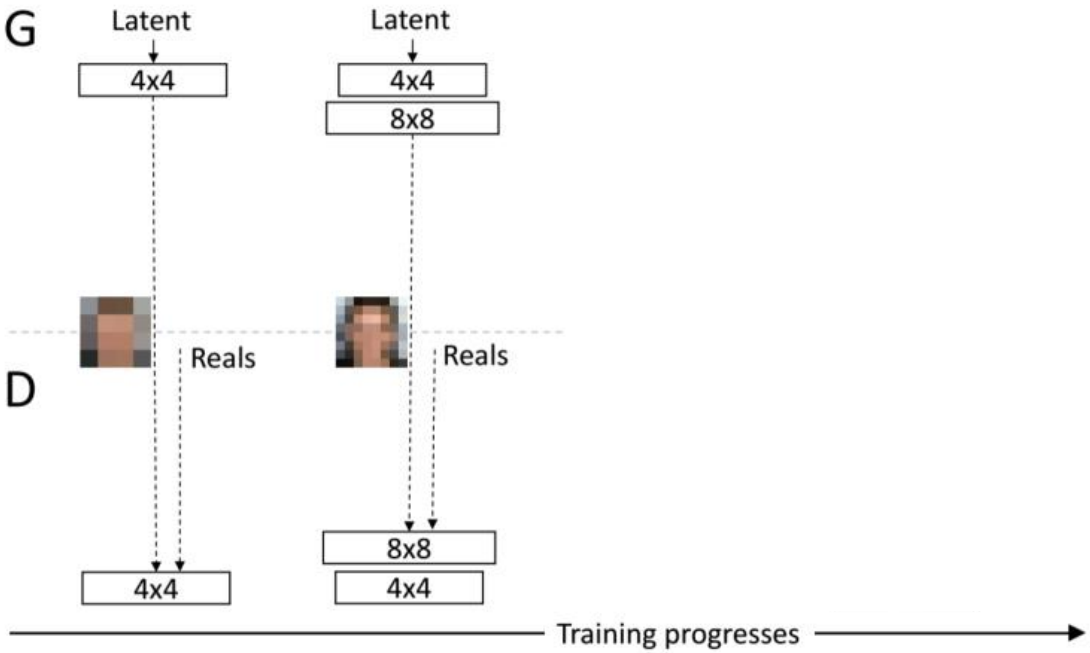
Low resolution images

[Karras et al., 2018]

Progressive GAN

Low resolution images

add in
additional
layers



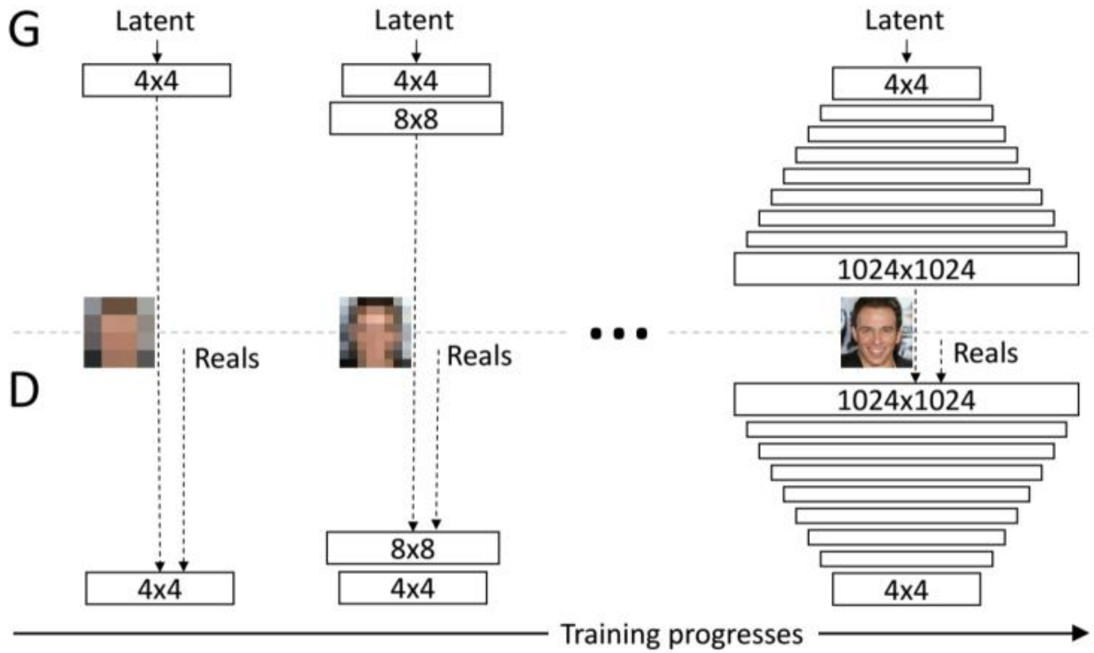
[Karras et al., 2018]

Progressive GAN

Low resolution images

add in additional layers

High resolution images



[Karras et al., 2018]

BigGAN

[Brock et al., 2018]

BigGAN

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- 2x – 4x more parameters
- 8x larger batch size
- Simple architecture changes that improve scalability

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BigGAN

- GANs benefit dramatically from scaling
- 2x
- 8x
- Sim

[Brock et al., 2018]

Key Takeaways

- Deep Generative Models: brief history

- GANs:
  - Implicit generative model
  - Minimax formulation
  - Wasserstein GAN

Questions?