DSC291: Machine Learning with Few Labels

Reinforcement Learning

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Lecture 23, May 29, 2024

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This Lecture

- RL (20mins)

- Presentation #1 (10mins):
  - Xujun Lian, Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks
- Presentation #2 (10mins):
  - Leo Chen, Learning Transferable Visual Models From Natural Language Supervision
- Presentation #3 (10mins):
  - Yiming Feng, Image Inpainting for Irregular Holes Using Partial Convolutions

Google form for presentation questions and feedback:

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
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Initialize replay memory, Q-network

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for episode $= 1, M$ **do** ← Play M episodes (full games)
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  for $t = 1, T$ **do**
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for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ ← Initialize state (starting game screen pixels) at the beginning of each episode
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
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  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do** ← For each timestep t of the game
    With probability ϵ select a random action a_t
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With small probability, select a random action (explore), otherwise select greedy action from current policy

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Take the action (a_t), and observe the reward r_t and next state s_{t+1}

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Store transition in replay memory

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Experience Replay: Sample a random minibatch of transitions from replay memory and perform a gradient descent step

Policy Gradients

What is a problem with Q-learning?
The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

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Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand
Can we learn a policy directly?

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right]$$

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Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Where $r(\tau)$ is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \ldots)$

REINFORCE algorithm

Expected reward:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} \left[r(\tau) \right]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

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$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$$
$$= \int_\tau r(\tau)p(\tau;\theta)\mathrm{d}\tau$$

Now let's differentiate this:

$$\nabla_\theta J(\theta) = \int_\tau r(\tau)\nabla_\theta p(\tau;\theta)\mathrm{d}\tau$$

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Intractable! Gradient of an expectation is problematic when p depends on θ

Question: How to estimate the gradient?

REINFORCE algorithm

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However, we can use a nice trick: $$\nabla_\theta p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_\theta \log p(\tau;\theta)$$

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If we inject this back:
$$\nabla_\theta J(\theta) = \int_{\tau} \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau$$
$$= \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\nabla_\theta \log p(\tau;\theta)\right]$$

Can estimate with Monte Carlo sampling

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

REINFORCE algorithm

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Thus: $$\log p(\tau;\theta) = \sum_{t\geq 0} \log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)$$

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And when differentiating: $$\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Doesn't depend on transition probabilities!

REINFORCE algorithm

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau$$
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Doesn't depend on transition probabilities!

Therefore when sampling a trajectory τ, we can estimate J(θ) with

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

Intuition

Gradient estimator: $$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Interpretation:

- If $r(\tau)$ is high, push up the probabilities of the actions seen
- If $r(\tau)$ is low, push down the probabilities of the actions seen

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Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

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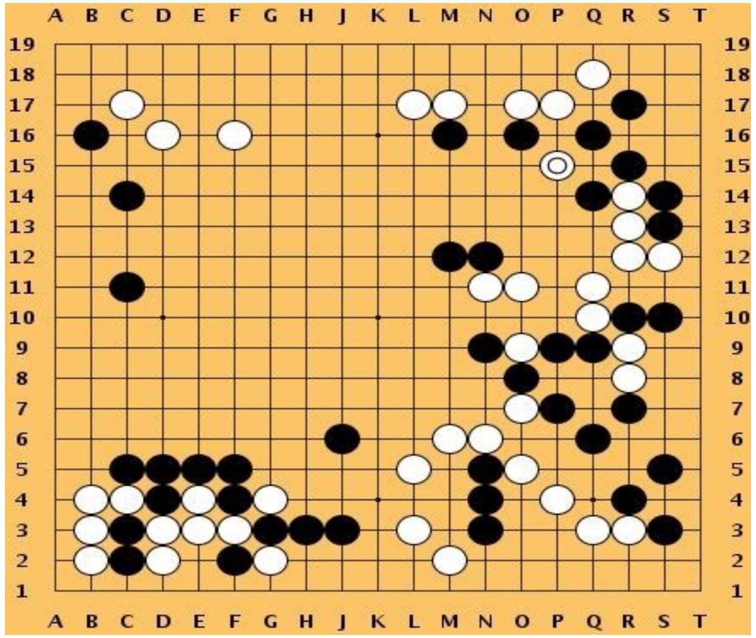
However, this also suffers from high variance because **credit assignment** is really hard.

More policy gradients: AlphaGo



Overview:

- Mix of supervised learning and reinforcement learning
- Mix of old methods (Monte Carlo Tree Search) and recent ones (deep RL)

How to beat the Go world champion:

- Featurize the board (stone color, move legality, bias, …)
- Initialize policy network with supervised training from professional go games, then continue training using policy gradient (play against itself from random previous iterations, +1 / -1 reward for winning / losing)
- Also learn value network (critic)
- Finally, combine policy and value networks in a Monte Carlo Tree Search algorithm to select actions by lookahead search

[Silver et al., Nature 2016]

Key Takeaways

- Markov Decision Process (MDP)
- Q-learning
  - Bellman equation
  - Deep Q-learning, experience replay
- Policy gradients

- Guarantees:
  - Policy Gradients: Converges to a local minima of J(θ), often good enough!
  - Q-learning: Zero guarantees since you are approximating Bellman equation with a complicated function approximator

Questions?