DSC250: Advanced Data Mining

Topic Models / Text Embedding

Zhiting Hu Lecture 9, Feb 4, 2025



Recap: Expectation Maximization (EM)

- Supervised MLE is easy:
 - \circ Observe both x and z
- Unsupervised MLE is hard:
 - \circ Observe only x
- EM, intuitively:

E-step:
$$q(\mathbf{z}|\mathbf{x}) = p(\mathbf{z}|\mathbf{x}, \theta)$$

M-step: $\max_{\theta} \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} [\log p(\mathbf{x}, \mathbf{z}|\theta)]$

 $\max_{\theta} \ell_c(\theta; \mathbf{x}, \mathbf{z}) = \log p(\mathbf{x}, \mathbf{z} | \theta)$

$$\max_{\theta} \ell(\theta; \mathbf{x}) = \log p(\mathbf{x}|\theta) = \log \sum_{\mathbf{z}} p(\mathbf{x}, \mathbf{z}|\theta)$$

We don't actually observe q, let's estimate it

Let's "pretend" we also observe **Z** (its distribution)

Recap: Expectation Maximization (EM)

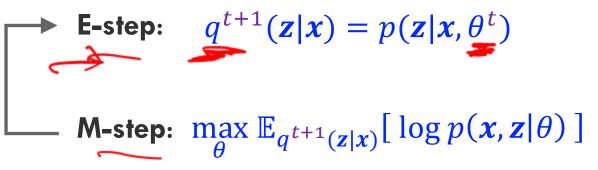
Supervised MLE is easy:

 $\max_{\theta} \ell_c(\theta; \mathbf{x}, \mathbf{z}) = \log p(\mathbf{x}, \mathbf{z} | \theta)$

- \circ Observe both x and z
- Unsupervised MLE is hard:

 $\max_{\theta} \ell(\theta; \mathbf{x}) = \log p(\mathbf{x}|\theta) = \log \sum_{\mathbf{z}} p(\mathbf{x}, \mathbf{z}|\theta)$

- \circ Observe only x
- EM, intuitively:



We don't actually observe q, let's estimate it

Let's "pretend" we also observe **Z** (its distribution)

This is an iterative process

Recap: Expectation Maximization (EM)

The EM algorithm is coordinate-decent on $F(q, \theta)$

$$\circ$$
 E-step: $q^{t+1} = \arg\min_{q} F\left(q, \theta^{t}\right) = p(\mathbf{z}|\mathbf{x}, \theta^{t})$

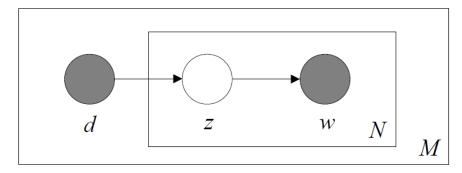
the posterior distribution over the latent variables given the data and the current parameters

$$\text{M-step:} \quad \theta^{t+1} = \arg\min_{\theta} F\left(q^{t+1}, \theta^{t}\right) = \arg\max_{\theta} \sum_{\mathbf{z}} q^{t+1}(\mathbf{z}|\mathbf{x}) \log p(\mathbf{x}, \mathbf{z}|\theta)$$

$$\ell(\theta; \mathbf{x}) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \left[\log \frac{p(\mathbf{x}, \mathbf{z}|\theta)}{q(\mathbf{z}|\mathbf{x})} \right] + \text{KL} \left(q(\mathbf{z}|\mathbf{x}) \mid\mid p(\mathbf{z}|\mathbf{x}, \theta) \right)$$

$$= -F(q, \theta) + \text{KL} \left(q(\mathbf{z}|\mathbf{x}) \mid\mid p(\mathbf{z}|\mathbf{x}, \theta) \right)$$

Recap: Learning pLSA with EM



Likelihood function of a word w:

$$p(w|d,\theta,\beta) = \sum_{k} p(w,z = k|d,\theta,\beta)$$

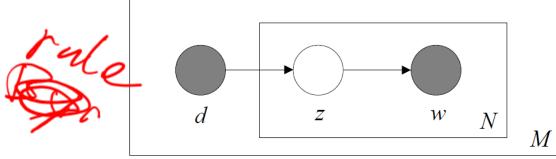
$$= \sum_{k} p(w|z = k,d, ,\beta)p(z = k|d,\theta,) = \sum_{k} \beta_{kw}\theta_{dk}$$

• Learning by maximizing the log likelihood:

Max log = Bkw Odp c

Bayes

Recap: Learning pLSA with ÉM



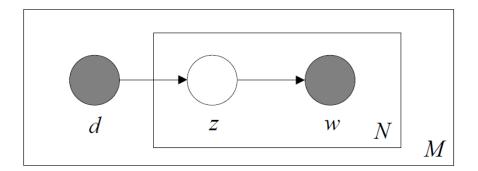
• E-step:

$$Q(2|w,d) = P(2|w,d,0^{\epsilon},\beta^{\epsilon}) = \int_{-\infty}^{\infty}$$

• M-step:

Max Eggand Cog PRW, d. of pe, 7 6

Recap: Learning pLSA with EM

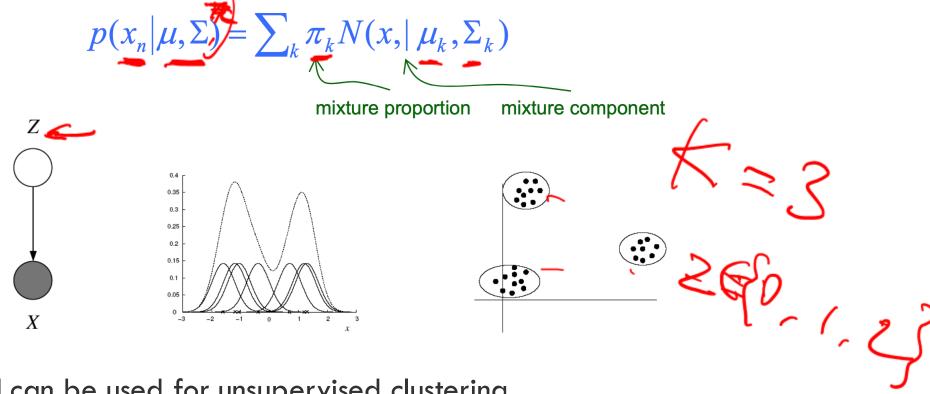


E-step:

$$p(z|w,d,\theta^t,\beta^t) = \frac{p(w|z,d,\beta^t)p(z|d,\theta^t)}{\sum_{z'}p(w|z',d,\beta^t)p(z'|d,\theta^t)} = \frac{\beta_{zw}^t\theta_{dz}^t}{\sum_{z'}\beta_{z'w}^t\theta_{dz'}^t}$$

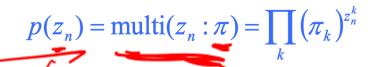
M-step:

Consider a mixture of K Gaussian components:



- This model can be used for unsupervised clustering.
 - This model (fit by AutoClass) has been used to discover new kinds of stars in astronomical data, etc.

- Consider a mixture of K Gaussian components:
 - Z is a latent class indicator vector:





$$p(x_n \mid z_n^k = 1, \mu, \Sigma) = \frac{1}{(2\pi)^{m/2} |\Sigma_k|^{1/2}} \exp\left\{-\frac{1}{2} (x_n - \mu_k)^T \Sigma_k^{-1} (x_n - \mu_k)\right\}$$

□ The likelihood of a sample:

$$P(X|X,U,Z) = \sum_{z} P(z|X) P(X|Z,U,Z)$$

$$Gos P(X|X,U,Z) = Gos Z - - -$$

- Consider a mixture of K Gaussian components:
 - Z is a latent class indicator vector:

$$p(z_n) = \text{multi}(z_n : \pi) = \prod_k (\pi_k)^{z_n^k}$$

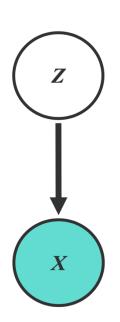


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The likelihood of a sample:

$$p(x_{n}|\mu,\Sigma) = \sum_{k} p(z^{k} = 1 | \pi) p(x, | z^{k} = 1, \mu, \Sigma)$$

$$= \sum_{z_{n}} \prod_{k} \left((\pi_{k})^{z_{n}^{k}} N(x_{n} : \mu_{k}, \Sigma_{k})^{z_{n}^{k}} \right) = \sum_{k} \pi_{k} N(x, | \mu_{k}, \Sigma_{k})$$



- Consider a mixture of K Gaussian components:
 - Z is a latent class indicator vector:

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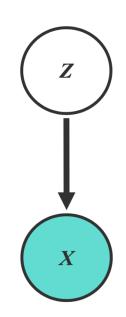
$$p(x_n \mid z_n^k = 1, \mu, \Sigma) = \frac{1}{(2\pi)^{m/2} |\Sigma_k|^{1/2}} \exp\left\{-\frac{1}{2} (x_n - \mu_k)^T \Sigma_k^{-1} (x_n - \mu_k)\right\}$$

The likelihood of a sample:

Parameters to be learned:

$$p(x_n|\mu, \Sigma) = \sum_k p(z^k = 1 \mid \pi) p(x, \mid z^k = 1, \mu, \Sigma)$$

$$= \sum_{z_n} \prod_k \left((\pi_k)^{z_n^k} N(x_n : \mu_k, \Sigma_k)^{z_n^k} \right) = \sum_k \pi_k N(x, \mid \mu_k, \Sigma_k)$$
mixture proportion
$$= \sum_k p(z^k = 1 \mid \pi) p(x, \mid z^k = 1, \mu, \Sigma)$$



mixture component

Consider a mixture of K Gaussian components

• E-step: computing the posterior of z_n given the current estimate of the parameters (i.e., π , μ , Σ)

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- Consider a mixture of K Gaussian components
- E-step: computing the posterior of z_n given the current estimate of the parameters (i.e., π , μ , Σ)

$$p(z_n^k = 1 | x, \mu^{(t)}, \Sigma^{(t)}) = \frac{\pi_k^{(t)} N(x_n, | \mu_k^{(t)}, \Sigma_k^{(t)})}{\sum_i \pi_i^{(t)} N(x_n, | \mu_i^{(t)}, \Sigma_i^{(t)})} p(x, \mu^{(t)}, \Sigma^{(t)})$$

- Consider a mixture of K Gaussian components
- E-step: computing the posterior of z_n given the current estimate of the parameters (i.e., π , μ , Σ)

$$p(z_n^k = 1 \mid x, \mu^{(t)}, \Sigma^{(t)}) = \frac{\pi_k^{(t)} N(x_n, | \mu_k^{(t)}, \Sigma_k^{(t)})}{\sum_i \pi_i^{(t)} N(x_n, | \mu_i^{(t)}, \Sigma_i^{(t)})} p(x_n^k = 1, x, \mu^{(t)}, \Sigma^{(t)})$$

M-step: the expected complete log likelihood

a, z, l $= 2(2(x, u^{\alpha}), z^{\alpha}) \left(\log P(z, x, l), z^{\alpha}\right) \left(\log P(z, x, l), z^{\alpha}\right)$

- Consider a mixture of K Gaussian components
- E-step: computing the posterior of z_n given the current estimate of the parameters (i.e., π , μ , Σ)

$$p(z_n^k = 1 \mid x, \mu^{(t)}, \Sigma^{(t)}) = \frac{\pi_k^{(t)} N(x_n, | \mu_k^{(t)}, \Sigma_k^{(t)})}{\sum_i \pi_i^{(t)} N(x_n, | \mu_i^{(t)}, \Sigma_i^{(t)})} p(x, \mu^{(t)}, \Sigma^{(t)})$$

M-step: the expected complete log likelihood

$$\mathbb{E}_{q} \left[\ell_{c}(\boldsymbol{\theta}; x, z) \right] = \sum_{n} \mathbb{E}_{q} \left[\log p \left(z_{n} \mid \pi \right) \right] + \sum_{n} \mathbb{E}_{q} \left[\log p \left(x_{n} \mid z_{n}, \mu, \Sigma \right) \right]$$

$$= \sum_{n} \sum_{k} \mathbb{E}_{q} \left[z_{n}^{k} \right] \log \pi_{k} - \frac{1}{2} \sum_{n} \sum_{k} \mathbb{E}_{q} \left[z_{n}^{k} \right] \left(\left(x_{n} - \mu_{k} \right)^{T} \Sigma_{k}^{-1} \left(x_{n} - \mu_{k} \right) + \log |\Sigma_{k}| + C \right)$$

• M-step: computing the parameters given the current estimate of \mathbb{Z}_n

$$\pi_{k}^{*} = \arg\max\langle l_{c}(\boldsymbol{\theta})\rangle, \qquad \Rightarrow \frac{\partial}{\partial \pi_{k}}\langle l_{c}(\boldsymbol{\theta})\rangle = 0, \forall k, \quad \text{s.t.} \sum_{k} \pi_{k} = 1$$

$$\Rightarrow \pi_{k}^{*} = \frac{\sum_{n}\langle z_{n}^{k}\rangle_{q^{(n)}}}{N} = \frac{\sum_{n}\tau_{n}^{k(t)}}{N} = \frac{\langle n_{k}\rangle}{N}$$

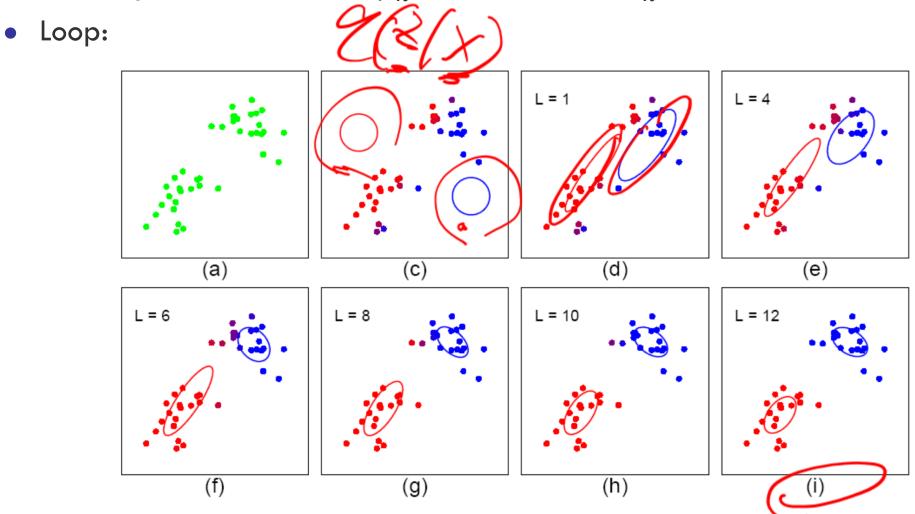
$$\mu_{k}^{*} = \arg\max\langle l(\boldsymbol{\theta})\rangle, \qquad \Rightarrow \mu_{k}^{(t+1)} = \frac{\sum_{n}\tau_{n}^{k(t)}x_{n}}{\sum_{n}\tau_{n}^{k(t)}}$$

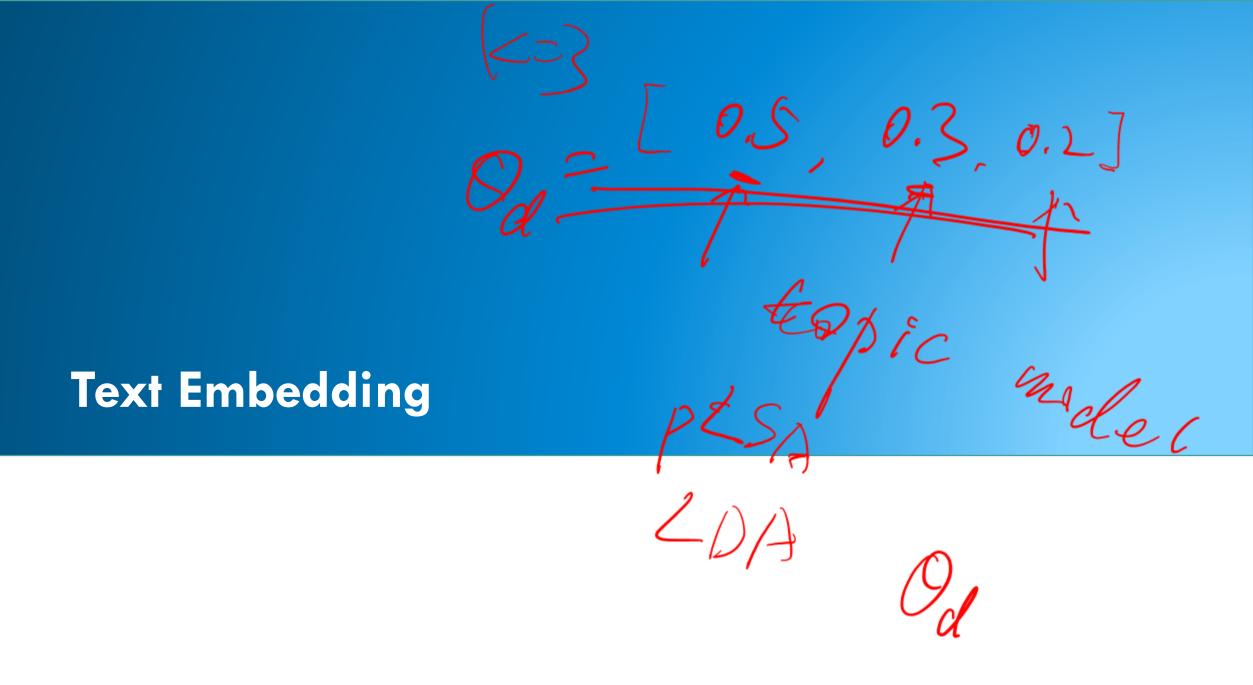
$$\Sigma_{k}^{*} = \arg\max\langle l(\boldsymbol{\theta})\rangle, \qquad \Rightarrow \Sigma_{k}^{(t+1)} = \frac{\sum_{n}\tau_{n}^{k(t)}(x_{n} - \mu_{k}^{(t+1)})(x_{n} - \mu_{k}^{(t+1)})^{T}}{\sum_{n}\tau_{n}^{k(t)}}$$

$$\frac{\partial \log|A^{-1}|}{\partial A^{-1}} = A^{T}$$

$$\frac{\partial \mathbf{x}^{T}A\mathbf{x}}{\partial A} = \mathbf{x}\mathbf{x}^{T}$$

• Start: "guess" the centroid μ_k and covariance Σ_k of each of the K clusters





Word Embedding Conventional word

embedding:

Word 2vec, Glove

A pre-trained **matrix**, each row is an embedding vector of a word

[Courtesy: <u>Vaswani</u>, et al., 2017][

-0.348680 0.841480 -0.773320 -0.2825400.580760 0.258540 -0.374120 -0.0762640.109260 0.186620 0.029943 0.182700 -0.631980 0.133060 -0.1289800.603430 0.171200 -0.348540 -0.0972340.295650 -0.516550 2.117200 0.534390 0.101800 -0.170860-0.041816 -0.260020 -0.3348400.215990 -0.3504400.411070 0.154010 -0.386110 0.206380 0.386700 1.460500 -0.215930 -0.035192 -0.126150 -0.6697400.513250 -0.797090-0.068611 -0.423290 -0.2645000.200870 0.082187 0.066944 1.027600 -0.989140 -0.259950 0.145960 0.766450 -0.303080 0.019587 -0.3549400.100180 -0.141530 -0.514270 0.886110 -0.530540 -0.016025 0.484620 -0.299200-0.353130 -0.3252900.156730 -0.430730 0.101390 0.761820 breakfast 0.073378 0.227670 0.208420 -0.456790 -0.078219 0.601960 -0.024494 -0.467980 0.054627 2.283700 0.130740 -0.193730 0.253270 0.090102 -0.272580-0.030571 0.096945 -0.115060 0.484000 0.848380 -0.1565700.594890 -0.031445 -0.0775860.278630 -0.509210 -0.066350 -0.081890 -0.047986 2.803600 0.129450 0.036518 0.032298 -0.0600340.399840 -0.103020-0.507880 0.076630 -0.422920 0.815730 -0.0723680.233200 0.137260 -0.156630 0.248440 0.349870 -0.241700 -0.091426 -0.530150 1.341300 0.259230 -0.8546900.360010 -0.6420000.568530 -0.3214200.173250 0.133030 -0.089720 1.528600 -0.048517 -0.0571200.007043 0.041856 -0.024704-0.039783 0.009614 0.308416 -0.174290-0.046976 0.287420 -0.128150 0.647630 0.056315 -0.240440 -0.025094 0.502220 -0.321940 -0.385640-0.353320-0.299710 -0.176230 0.586110 0.411160 -0.418680 0.073093 1.486500 -0.125650 0.048748 0.152440 0.199060 -0.065970 0.161950 0.212780 0.021170

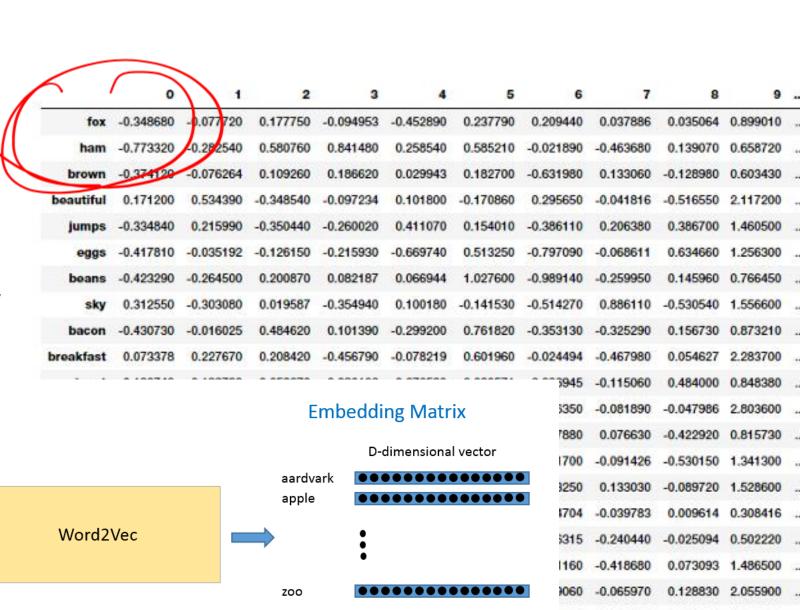
20 rows x 300 columns

Word Embedding

- Conventional word embedding:
 - Word2vec, Glove
 - A pre-trained matrix, each row is an embedding vector of a word

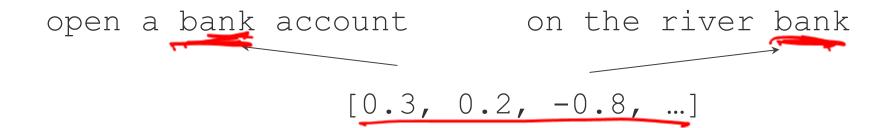
English Wikipedia Corpus

The Annual Reminder continued through July 4, 1969. This final Annual Reminder took place less than a week after the June 28 Stonewall riots, in which the patrons of the Stonewall Inn, a gay bar in Greenwich Village, fought against police who raided the bar. Rodwell received several telephone calls threatening him and the other New York participants, but he was able to arrange for police protection for the chartered bus all the way to Philadelphia. About 45 people participated, including the deputy mayor of Philadelphia and his wife. The dress code was still in effect at the Reminder, but two women from the New York contingent broke from the single-file picket line and held hands. When Kameny tried to break them apart, Rodwell furiously denounced him to onlooking members of the press.



Word Embedding

• Problem: word embeddings are applied in a confext free manner



Courtesy: Devlin 2019

Word Embedding

• Problem: word embeddings are applied in a context free manner

• Solution: Train contextual representations on text corpus

$$[0.9, -0.2, 1.6, ...]$$
 $[-1.9, -0.4, 0.1, ...]$ open a bank account on the river bank

Courtesy: Devlin 2019

Contextual Representations

GPT &

Improving Language Understanding by Generative Pre-Training,
 OpenAI, 2018

Train Deep (12-layer) Transformer LM Fine-tune on Classification Task POSITIVE Transformer Transformer

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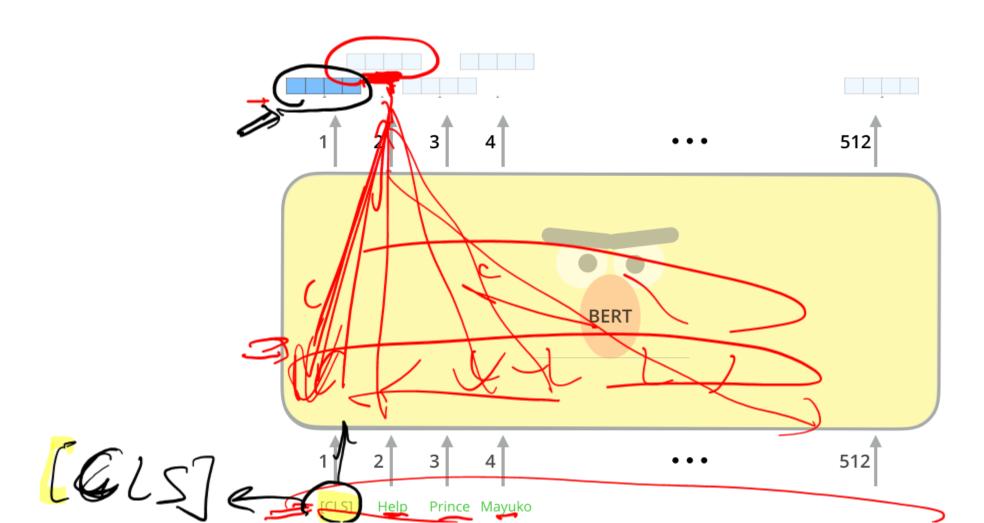
Problem with Previous Methods

• **Problem**: Language models only use left context or right context, but language understanding is bidirectional.

courtesy: Devlin 2019

BERT

Pretrainsing • BERT: A bidirectional model to extract contextual word embedding

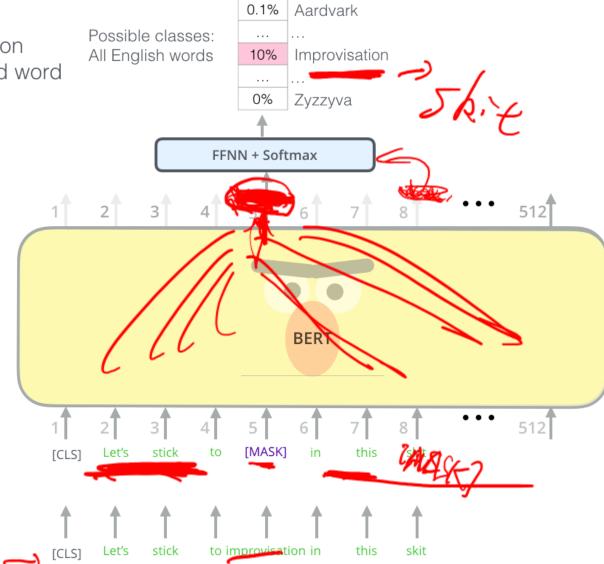


- Dataset:
 - Wikipedia (2.5B words) + a collection of free ebooks (800M words)

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- Training procedure
 - masked language model (masked LM)
 - Masks some percent of words from the input and has to reconstruct those words from context

Masked LM

Use the output of the masked word's position to predict the masked word



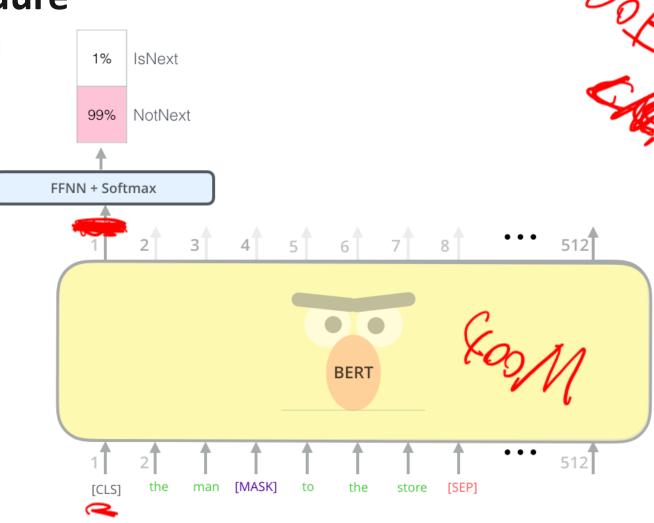
Randomly mask 15% of tokens

Input

- Masked LM
- 15% masking:
 - Too little masking: Too expensive to train (few supervision signals per example)
 - Too much masking: Not enough context
- Problem: Mask token never seen at fine-tuning
- Solution: don't replace with [MASK] 100% of the time. Instead:
- 80% of the time, replace with [MASK]
 - \circ went to the store \rightarrow went to the [MASK]
- 10% of the time, replace random word
 - went to the store → went to the running
- 10% of the time, keep same
 - went to the store → went to the store

- Dataset:
 - Wikipedia (2.5B words) + a collection of free ebooks (800M words)
- Training procedure
 - masked language model (masked LM)
 - Masks some percent of words from the input and has to reconstruct those words from context
 - Two-sentence task
 - To understand relationships between sentences
 - Concatenate two sentences A and B and predict whether B actually comes after A in the original text

Two sentence task Predict likelihood that sentence B belongs after sentence A



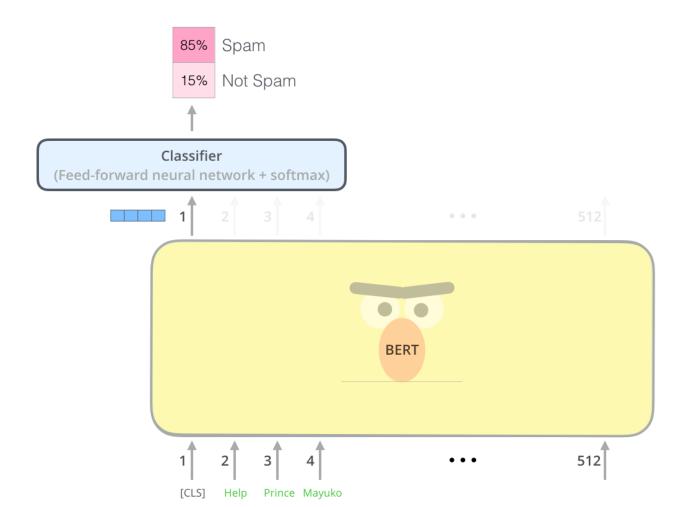


[CLS] the man [MASK] to the store [SEP] penguin [MASK] are flightless birds [SEP]

Sentence A Sentence B

BERT: Downstream Fine-tuning

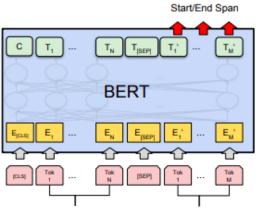
• Use BERT for sentence classification



BERT: Downstream Fine-tuning

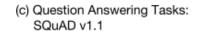
BERT Sentence 1 (a) Sentence Pair Classification Tasks: MNLI, QQP, QNLI, STS-B, MRPC, 🗖 RTE, SWAG

Label

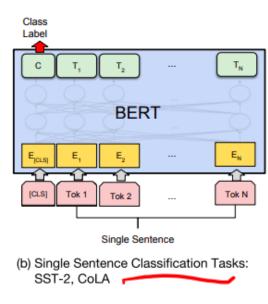


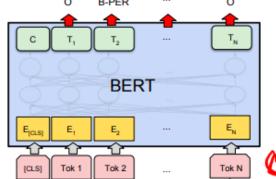
Sentence 2

Paragraph



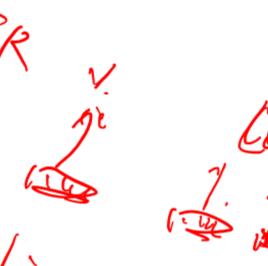
Question













(d) Single Sentence Tagging Tasks: CoNLL-2003 NER

Single Sentence

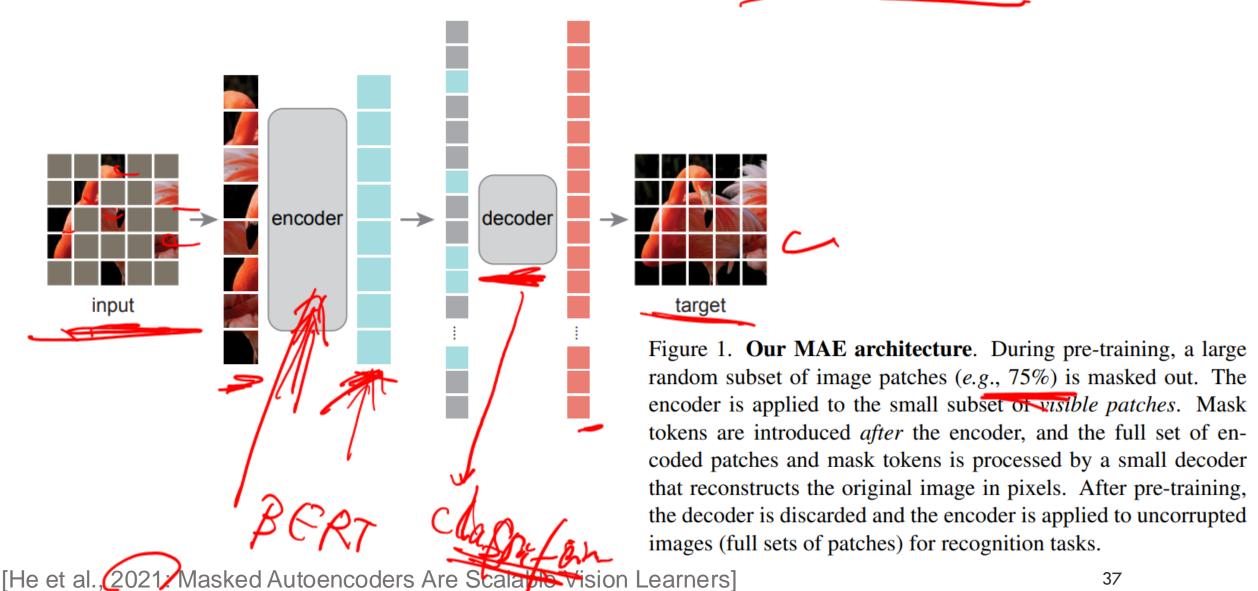
BERT Results

Huge improvements over SOTA on 12 NLP task

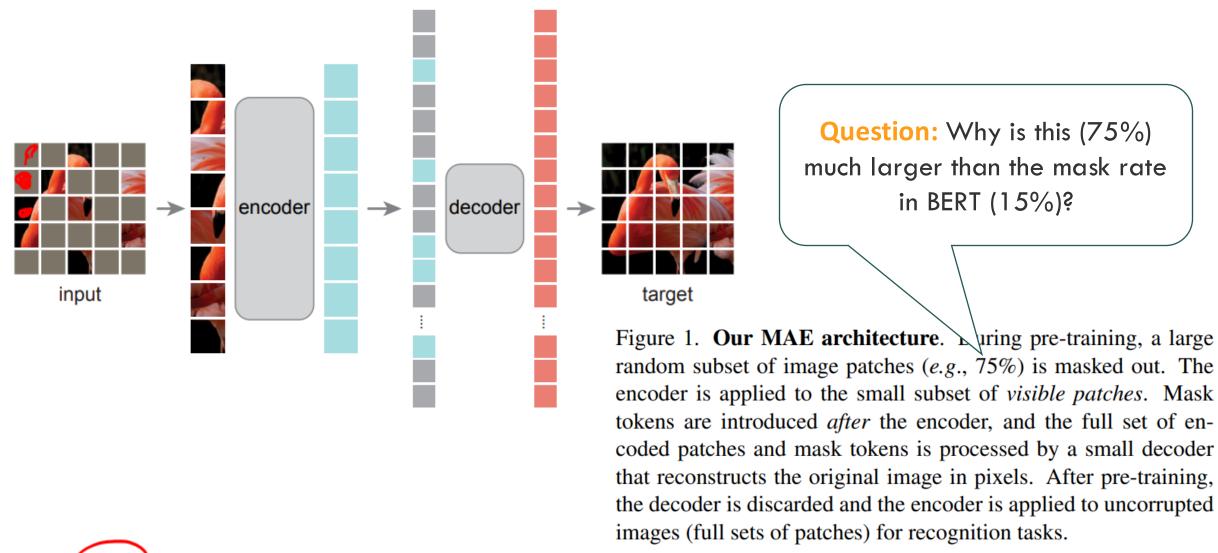
				· /					
System	MNLI-(m/mm)	QQP	QNLI	SST-2	CoLA	STS-B	MRPC	RTE	Average
	392k	363k	108k	67k	8.5k	5.7k	3.5k	2.5k	_
Pre-OpenAI SOTA	80.6/80.1	66.1	82.3	93.2	35.0	81.0	86.0	61.7	74.0
BiLSTM+ELMo+Attn	76.4/76.1	64.8	79.9	90.4	36.0	73.3	84.9	56.8	71.0
OpenAI GPT	82.1/81.4	70.3	88.1	91.3	45.4	80.0	82.3	56.0	75.2
BERT _{BASE}	84.6/83.4	71.2	90.1	93.5	52.1	85.8	88.9	66.4	79.6
$BERT_{LARGE}$	86.7/85.9	72.1	91.1	94.9	60.5	86.5	89.3	70.1	81.9

Table 1: GLUE Test results, scored by the GLUE evaluation server. The number below each task denotes the number of training examples. The "Average" column is slightly different than the official GLUE score, since we exclude the problematic WNLI set. OpenAI GPT = (L=12, H=768, A=12); BERT_{BASE} = (L=12, H=768, A=12); BERT_{LARGE} = (L=24, H=1024, A=16). BERT and OpenAI GPT are single-model, single task. All results obtained from https://gluebenchmark.com/leaderboard and https://blog.openai.com/language-unsupervised/.

Similar idea for image embedding: masked autoencoder (MAE)



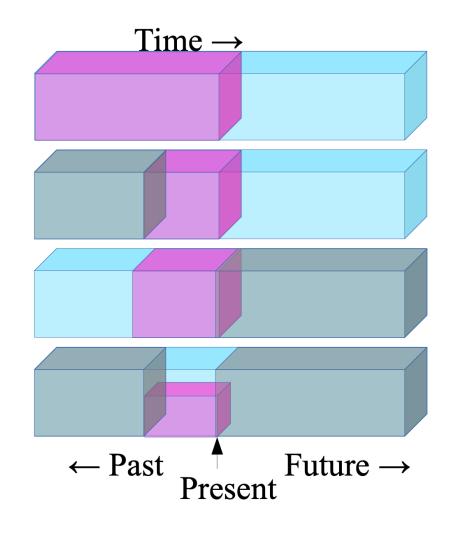
Similar idea for image embedding: masked autoencoder (MAE)



More general idea: Self-Supervised Learning

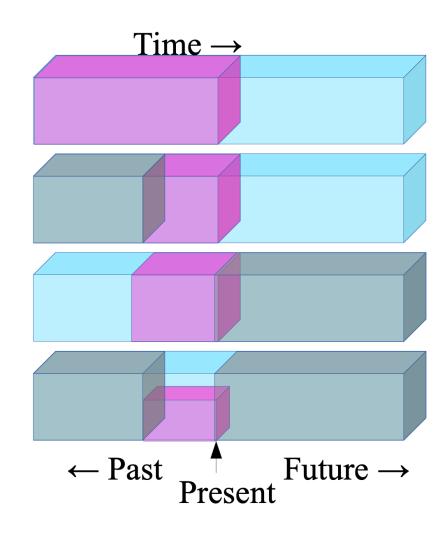
- Predict any part of the input from any other part.
- **▶** Predict the future from the past.
- **▶** Predict the future from the recent past.
- Predict the past from the present.
- Predict the top from the bottom.





More general idea: Self-Supervised Learning

- Predict any part of the input from any other part.
- Predict the future from the past.
- ► Predict the future from the recent past.
- Predict the past from the present.
- Predict the top from the bottom.
- ► Predict the <u>occluded from the visible</u>
- Pretend there is a part of the input you don't know and predict that.



More general idea: Self-Supervised Learning: Motivation (I)

Our brains do this all the time

- Filling in the visual field at the retinal blind spot
- Filling in occluded images, missing segments in speech
- Predicting the state of the world from partial (textual) descriptions
- Predicting the consequences of our actions
- Predicting the sequence of actions leading to a result





More general idea: Self-Supervised Learning: Motivation (I)

- Successfully learning to predict everything from everything else would result in the accumulation of lots of background knowledge about how the world works
- The model is forced to learn what we really care about, e.g. a semantic representation, in order to solve the prediction problem

[Courtesy: Lecun "Self-supervised Learning"]

[Courtesy: Zisserman "Self-supervised Learning"]

More general idea: Self-Supervised Learning: Motivation (II)

- The machine predicts any part of its input from any observed part
 - A lot of supervision signals in each data instance
- Untapped/availability of vast numbers of unlabeled text/images/videos...
 - Facebook: one billion images uploaded per day
 - 300 hours of video are uploaded to YouTube every minute

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X Closed of the

Questions?