DSC250: Advanced Data Mining

Topic Models

Zhiting Hu Lecture 6, Jan 24, 2025



In-class paper presentation

Adversarial Examples are not Bugs, they are Features



Andrew Ilyas*



Dimitris Tsipras*



Shibani Santurkar*



Logan Engstrom*



Brandon Tran



Aleksander Mądry

Topic Models

Outline

- Representations of Text and Topics
- Topic Model v1: Multinomial Mixture Model
- Topic Model v2: Probabilistic Latent Semantic Analysis (pLSA)
- Topic Model v3: Latent Dirichlet Allocation (LDA)

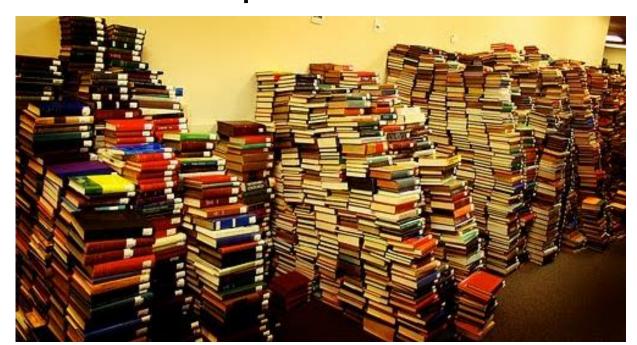
Slides adapted from:

- Y. Sun, CS 247: Advanced Data Mining
- M. Gormley, 10-701 Introduction to Machine Learning

Motivation

Suppose'you're'given'a'massive'corpora'and'asked'to'carry'but'the' following'tasks"

- Organize'the'documents'Into'thematic&ategories&
- **Describe**'the 'evolution' bf'those 'categories' **bver&ime**&
- Enable'a'domain'expert'to'analyze&nd&nderstand&he'content"
- Find'relationships'between'the'categories"
- Understandthow'authorship'Influnces "the" ont ent "



Motivation

Suppose you're given a massive corpora and asked to carry out the following tasks

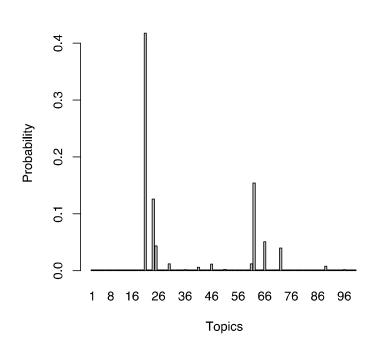
- Organize the documents into thematic categories
- Describe the evolution of those categories over time
- Enable a domain expert to analyze and understand the content
- Find relationships between the categories
- Understand how authorship influences the content

Topic Modeling:

A method of (usually unsupervised) discovery of latent or hidden structure in a corpus

- Applied primarily to text corpora, but techniques are more general
- Provides a modeling toolbox
- Has prompted the exploration of a variety of new inference methods to accommodate large-scale datasets

Topic Modeling: Examples



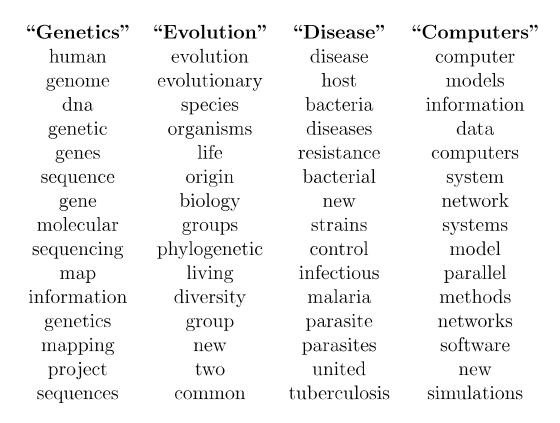
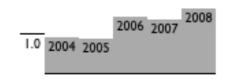


Figure from (Blei, 2011), shows topics and top words learned automatically from reading 17,000 Science articles

Topic Modeling: Examples

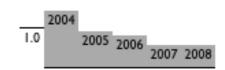
Dirichlet3multinomial&egression&DMR)&opic&nodel&n&CML& (Mimno'&'McCallum,'2008)"

Topic 0 [0.152]



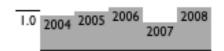
problem, optimization, problems, convex, convex optimization, linear, semidefinite programming, formulation, sets, constraints, proposed, margin, maximum margin, optimization problem, linear programming, programming, procedure, method, cutting plane, solutions

Topic 54 [0.051]



decision trees, trees, tree, decision tree, decision, tree ensemble, junction tree, decision tree learners, leaf nodes, arithmetic circuits, ensembles modts, skewing, ensembles, anytime induction decision trees, trees trees, random forests, objective decision trees, tree learners, trees grove, candidate split

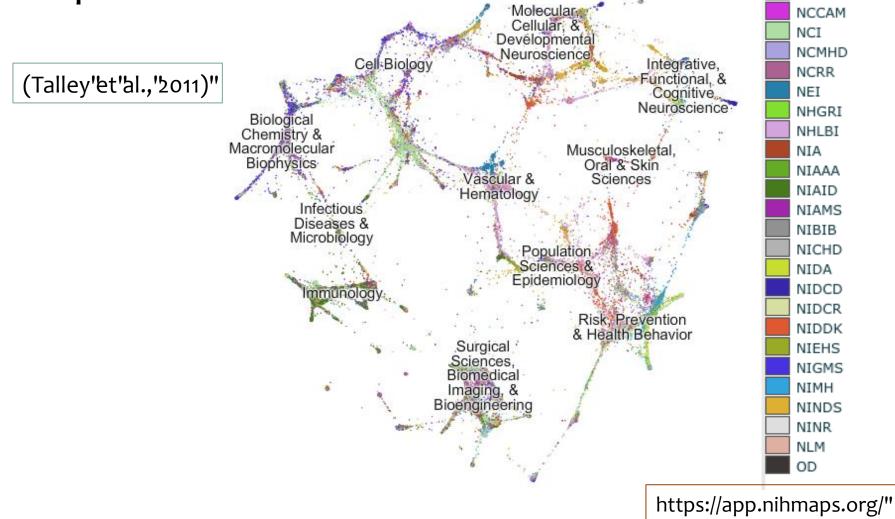
Topic 99 [0.066]



inference, approximate inference, exact inference, markov chain, models, approximate, gibbs sampling, variational, bayesian, variational inference, variational bayesian, approximation, sampling, methods, exact, bayesian inference, dynamic bayesian, process, mcmc, efficient http:///www.cs.umass.edu/emimpo/icm

Topic Modeling: Examples

Map'bf'NIH'Grants"

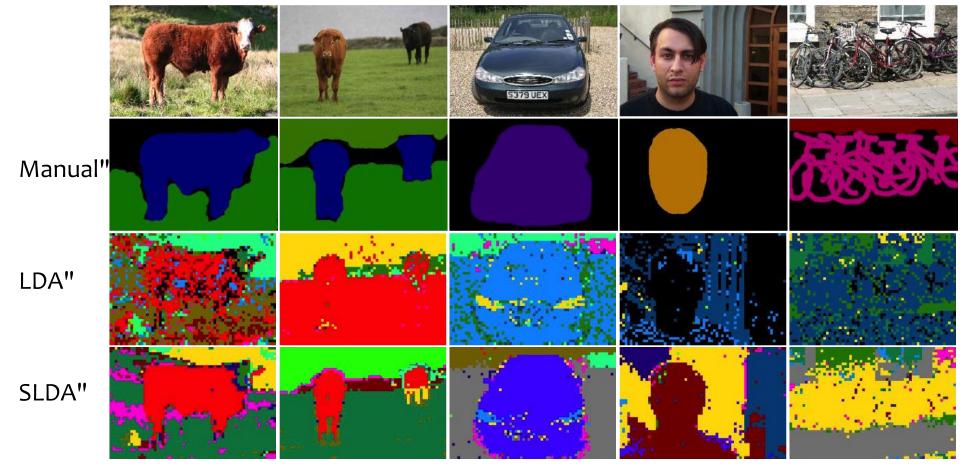


FIC

Other Applications of Topic Models

Spacial'LDA"

(Wang'&'Grimson,'2007)"



Other Applications of Topic Models

Word Sense Induction

Senses of drug (WSJ)

- 1. U.S., administration, federal, against, war, dealer
- 2. patient, people, problem, doctor, company, abuse
- 3. company, million, sale, maker, stock, inc.
- 4. administration, food, company, approval, FDA

(Brody & Lapata, 2009)

Senses of *drug* (BNC)

- 1. patient, treatment, effect, anti-inflammatory
- 2. alcohol, treatment, patient, therapy, addiction
- 3. patient, new, find, effect, choice, study
- 4. test, alcohol, patient, abuse, people, crime
- 5. trafficking, trafficker, charge, use, problem
- 6. abuse, against, problem, treatment, alcohol
- 7. people, wonder, find, prescription, drink, addict

• Selectional Preference 8. company, dealer, police, enforcement, patient

(Ritter et al., 2010)

| Topic t | Arg1 | Relations which assign | Arg2 |
|---------|--|----------------------------|--|
| | | highest probability to t | |
| 18 | The residue - The mixture - The reaction | was treated with, is | EtOAc - CH2Cl2 - H2O - CH.sub.2Cl.sub.2 |
| | mixture - The solution - the mixture - the re- | treated with, was | - H.sub.2O - water - MeOH - NaHCO3 - |
| | action mixture - the residue - The reaction - | poured into, was | Et2O - NHCl - CHCl.sub.3 - NHCl - drop- |
| | the solution - The filtrate - the reaction - The | extracted with, was | wise - CH2Cl.sub.2 - Celite - Et.sub.2O - |
| | product - The crude product - The pellet - | purified by, was di- | Cl.sub.2 - NaOH - AcOEt - CH2C12 - the |
| | The organic layer - Thereto - This solution | luted with, was filtered | mixture - saturated NaHCO3 - SiO2 - H2O |
| | - The resulting solution - Next - The organic | through, is disolved in, | - N hydrochloric acid - NHCl - preparative |
| | phase - The resulting mixture - C.) | is washed with | HPLC - to0 C |

Text Data

- Word/term
- Document
 - A sequence of words
- Corpus
 - A collection of documents



Represent a Document

- Most common way: Bag-of-Words
 - Ignore the order of words
 - keep the count
- c1: Human machine interface for Lab ABC computer applications
- c2: A survey of user opinion of computer system response time
- c3: The EPS user interface management system
- c4: System and human system engineering testing of EPS
- c5: Relation of user-perceived response time to error measurement
- m1: The generation of random, binary, unordered trees
- m2: The intersection graph of paths in trees
- m3: Graph minors IV: Widths of trees and well-quasi-ordering
- m4: Graph minors: A survey

| | | c1 | c2 | c3 | c4 | c5 | m1 | m2 | m3 | m4 |
|---|-----------|----|----|----|----|----|----|----|----|----|
| | human | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | interface | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| | computer | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | user | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| | system | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| / | response | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | time | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | EPS | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| | survey | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| | trees | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| | graph | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | minors | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| | | | | | | | | | | |

Represent a Document

- Represent the doc as a vector where each entry corresponds to a different word and the number at that entry corresponds to how many times that word was present in the document (or some function of it)
 - Number of words is huge
 - Select and use a smaller set of words that are of interest
 - E.g. uninteresting words: 'and', 'the' 'at', 'is', etc. These are called <u>stop-words</u>
 - Stemming: remove endings. E.g. 'learn', 'learning', 'learnable', 'learned' could be substituted by the single stem 'learn'
 - Other simplifications can also be invented and used
 - The set of different remaining words is called <u>dictionary</u> or <u>vocabulary</u>. Fix an ordering of the terms in the dictionary so that you can operate them by their index.
 - Can be extended to bi-gram, tri-gram, or so

Limitations of Bag-of-Words

- Dimensionality
 - High dimensionality
- Sparseness
 - Most of the entries are zero
- Shallow representation
 - The vector representation does not capture semantic relations between words

Ex: "Tom loves Kate."

Represent a Topic

- A topic is represented by a word distribution
- Relate to an issue

| universe | 0.0439 | drug | 0.0672 | cells | 0.0675 | sequence | 0.0818 | years | 0.156 |
|------------|--------|--------------|---------|------------|---------|------------|--------|-----------|---------|
| galaxies | 0.0375 | patients | 0.0493 | stem | 0.0478 | sequences | 0.0493 | million | 0.0556 |
| clusters | 0.0279 | drugs | 0.0444 | human | 0.0421 | genome | 0.033 | ago | 0.045 |
| matter | 0.0233 | clinical | 0.0346 | cell | 0.0309 | dna | 0.0257 | time | 0.0317 |
| galaxy | 0.0232 | treatment | 0.028 | gene | 0.025 | sequencing | 0.0172 | age | 0.0243 |
| cluster | 0.0214 | trials | 0.0277 | tissue | 0.0185 | map | 0.0123 | year | 0.024 |
| cosmic | 0.0137 | therapy | 0.0213 | cloning | 0.0169 | genes | 0.0122 | record | 0.0238 |
| dark. | 0.0131 | trial | 0.0164 | transfer | 0.0155 | chromosome | 0.0119 | early | 0.0233 |
| light | 0.0109 | disease | 0.0157 | blood | 0.0113 | regions | 0.0119 | billion | 0.0177 |
| density | 0.01 | medical | 0.00997 | embryos | 0.0111 | human | 0.0111 | history | 0.0148 |
| bacteria | 0.0983 | male | 0.0558 | theory | 0.0811 | immune | 0.0909 | stars | 0.0524 |
| bacterial | 0.0561 | females | 0.0541 | physics | 0.0782 | response | 0.0375 | star | 0.0458 |
| resistance | 0.0431 | female | 0.0529 | physicists | 0.0146 | system | 0.0358 | astrophys | 0.0237 |
| coli | 0.0381 | males | 0.0477 | einstein | 0.0142 | responses | 0.0322 | mass | 0.021 |
| strains | 0.025 | sex | 0.0339 | university | 0.013 | antigen | 0.0263 | disk | 0.0173 |
| microbiol | 0.0214 | reproductive | 0.0172 | gravity | 0.013 | antigens | 0.0184 | black | 0.0161 |
| microbial | 0.0196 | offspring | 0.0168 | black | 0.0127 | immunity | 0.0176 | gas | 0.0149 |
| strain | 0.0165 | sexual | 0.0166 | theories | 0.01 | immunology | 0.0145 | stellar | 0.0127 |
| salmonella | 0.0163 | reproduction | 0.0143 | aps | 0.00987 | antibody | 0.014 | astron | 0.0125 |
| resistant | 0.0145 | eggs | 0.0138 | matter | 0.00954 | autoimmune | 0.0128 | hole | 0.00824 |

CODIC 42

TOPIC 47 Mayand approver active control of the cont

OPIC 45

constant identified installed figurery annually implement of the more annually of the

TOPIC 4

movalused mechanisms feader crimits mount on the property of t

TOPIC 43

remain and the second s

TOPIC 46

The poliments in the process of the policy o

TOPIC 49

productions of the production of the production

governing required indivipations for the priorities bring re ensuring en reconit

Trail

Topic Models

- Topic modeling
 - Get topics automatically from a corpus
 - Assign documents to topics automatically
- Most frequently used topic models
 - pLSA
 - LDA

| "Arts" | "Budgets" | "Children" | "Education" |
|--|--|---|--|
| NEW FILM SHOW MUSIC MOVIE PLAY MUSICAL BEST ACTOR FIRST YORK OPERA | "Budgets" MILLION TAX PROGRAM BUDGET BILLION FEDERAL YEAR SPENDING NEW STATE PLAN MONEY | "Children" CHILDREN WOMEN PEOPLE CHILD YEARS FAMILIES WORK PARENTS SAYS FAMILY WELFARE MEN | "Education" SCHOOL STUDENTS SCHOOLS EDUCATION TEACHERS HIGH PUBLIC TEACHER BENNETT MANIGAT NAMPHY STATE |
| THEATER ACTRESS LOVE | PROGRAMS GOVERNMENT CONGRESS | PERCENT CARE LIFE | PRESIDENT ELEMENTARY HAITI |
| | | | |

The William Randolph Hearst Foundation will give \$1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be \$200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive \$400,000 each. The Juilliard School, where music and the performing arts are taught, will get \$250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual \$100,000 donation, too.

Notations

- Word, document, topic
 - \circ w, d, z
- Word count in document:
 - \circ c(w,d): number of times word w occurs in document d
 - \circ or x_{dn} : number of times the nth word in the vocabulary occurs in document d
- Word distribution for each topic (β_z)
 - \circ β_{zw} : p(w|z)



Recap: Multinomial distribution

- Multinomial distribution
 - O Discrete random variable x that takes one of M values $\{1, ..., M\}$
 - $\circ p(\mathbf{x}=i) = \pi_i, \qquad \sum_i \pi_i = 1$
 - \circ Out of n independent trials, let k_i be the number of times x=i was observed
 - The probability of observing a vector of occurrences $\mathbf{k} = [k_1, ..., k_M]$ is given by the multinomial distribution parametrized by π

$$p(\mathbf{k}|\boldsymbol{\pi}, \mathbf{n}) = p(k_1, \dots, k_m | \pi_1, \dots, \pi_m, \mathbf{n}) = \frac{n!}{k_1! k_2! \dots k_m!} \prod_{i=1}^{n} \pi_i^{k_i}$$

- E.g., describing a text document by the frequency of occurrence of every distinct word
- \circ For n=1, a.k.a. categorical distribution
 - $p(\mathbf{x} = i \mid \boldsymbol{\pi}) = \pi_i$
 - In $k = [k_1, ..., k_M]$: $k_i = 1$, and $k_j = 0$ for all $j \neq i \rightarrow a.k.a.$, one-hot representation of i

- For documents with bag-of-words representation
 - $x_d = (x_{d1}, x_{d2}, ..., x_{dN}), x_{dn}$ is the number of words for nth word in the vocabulary
- Generative model

- For documents with bag-of-words representation
 - $x_d = (x_{d1}, x_{d2}, ..., x_{dN}), x_{dn}$ is the number of words for nth word in the vocabulary
- Generative model

Formulating the statistical relationship between words, documents and latent topics as a generative process describing how documents are created

- For documents with bag-of-words representation
 - $x_d = (x_{d1}, x_{d2}, ..., x_{dN}), x_{dn}$ is the number of words for nth word in the vocabulary
- Generative model
 - For each document
 - Sample its cluster label $z\sim Categorical(\pi)$
 - $\pi = (\pi_1, \pi_2, ..., \pi_K)$, π_k is the proportion of jth cluster
 - $p(z=k)=\pi_k$
 - Sample its word vector $x_d \sim multinomial(\beta_z)$
 - $\pmb{\beta}_z=(\beta_{z1},\beta_{z2},...,\beta_{zN}),\beta_{zn}$ is the parameter associate with nth word in the vocabulary

•
$$p(\mathbf{x}_d|z=k) = \frac{(\sum_n x_{dn})!}{\prod_n x_{dn}!} \prod_n \beta_{kn}^{x_{dn}} \propto \prod_n \beta_{kn}^{x_{dn}}$$

Graphical Model

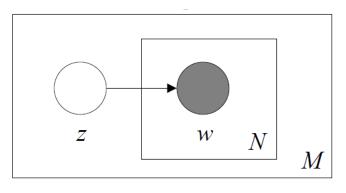
- Plates indicate replicated variables.
- Shaded nodes are observed; unshaded nodes are hidden.

Generative model

- For each document
 - Sample its cluster label $z\sim Categorical(\pi)$
 - $\pi = (\pi_1, \pi_2, ..., \pi_K)$, π_k is the proportion of jth cluster
 - $p(z=k)=\pi_k$
 - Sample its word vector $x_d \sim multinomial(\beta_z)$
 - $\beta_z=(\beta_{z1},\beta_{z2},...,\beta_{zN}),\beta_{zn}$ is the parameter associate with nth word in the vocabulary

•
$$p(\mathbf{x}_d|z=k) = \frac{(\sum_n x_{dn})!}{\prod_n x_{dn}!} \prod_n \beta_{kn}^{x_{dn}} \propto \prod_n \beta_{kn}^{x_{dn}}$$

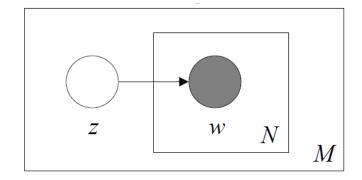
Graphical Model



- Plates indicate replicated variables.
- Shaded nodes are observed; unshaded nodes are hidden.

- Generative model
 - For each document
 - Sample its cluster label $z\sim Categorical(\pi)$
 - $\pi = (\pi_1, \pi_2, ..., \pi_K)$, π_k is the proportion of jth cluster
 - $p(z=k)=\pi_k$
 - Sample its word vector $x_d \sim multinomial(\beta_z)$
 - $\beta_z = (\beta_{z1}, \beta_{z2}, ..., \beta_{zN}), \beta_{zn}$ is the parameter associate with nth word in the vocabulary

•
$$p(\mathbf{x}_d|z=k) = \frac{(\sum_n x_{dn})!}{\prod_n x_{dn}!} \prod_n \beta_{kn}^{x_{dn}} \propto \prod_n \beta_{kn}^{x_{dn}}$$



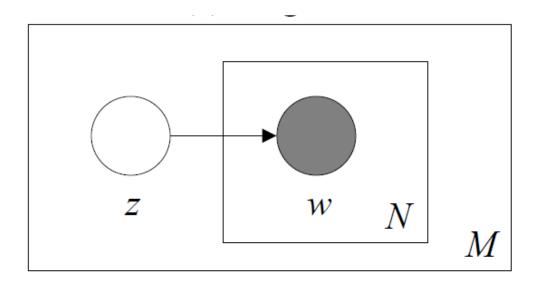
$$L = \prod_{d} p(\mathbf{x}_d) = \prod_{d} \sum_{k} p(\mathbf{x}_d, z = k)$$

$$= \prod_{d} \sum_{k} p(\mathbf{x}_d | z = k) p(z = k)$$

$$= \prod_{d} \frac{(\sum_{n} x_{dn})!}{\prod_{n} x_{dn}!} \sum_{k} p(z = k) \prod_{n} \beta_{kn}^{x_{dn}}$$

Limitations of Multinomial Mixture Model

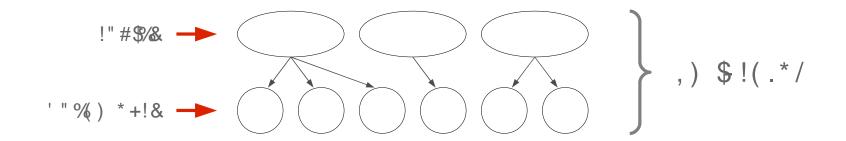
• All the words in the same documents are sampled from the same topic



• In practice, people switch topics during their writing

Limitations of Multinomial Mixture Model

Mixture'vs.'Admixture'



Topic Model v2: Probabilistic Latent Semantic Analysis (pLSA)

| "Arts" | "Budgets" | "Children" | "Education" | |
|---------|-------------------------|--------------|-------------|--|
| | | | | |
| NEW | MILLION | CHILDREN | SCHOOL | |
| FILM | TAX | WOMEN | STUDENTS | |
| SHOW | PROGRAM | PEOPLE | SCHOOLS | |
| MUSIC | BUDGET | CHILD | EDUCATION | |
| MOVIE | BILLION | YEARS | TEACHERS | |
| PLAY | FEDERAL | FAMILIES | HIGH | |
| MUSICAL | YEAR | WORK | PUBLIC | |
| BEST | SPENDING | PARENTS | TEACHER | |
| ACTOR | NEW | SAYS | BENNETT | |
| FIRST | STATE | FAMILY | MANIGAT | |
| YORK | PLAN | WELFARE | NAMPHY | |
| OPERA | MONEY | MEN | STATE | |
| THEATER | PROGRAMS | PERCENT | PRESIDENT | |
| ACTRESS | GOVERNMENT | CARE | ELEMENTARY | |
| LOVE | CONGRESS | $_{ m LIFE}$ | HAITI | |
| | | | | |

The William Randolph Hearst Foundation will give \$1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be \$200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive \$400,000 each. The Juilliard School, where music and the performing arts are taught, will get \$250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual \$100,000 donation, too.

Generative Model for pLSA

- For each position in d, $n = 1, ..., N_d$
 - Generate the topic for the position as

$$z_n|d\sim Categorical(\boldsymbol{\theta}_d), i.e., p(z_n=k|d)=\theta_{dk}$$
 (Note, 1 trial multinomial)

Generate the word for the position as

$$w_n|z_n\sim Categorical(\boldsymbol{\beta}_{z_n})$$
, i.e., $p(w_n=w|z_n)=\beta_{z_nw}$

Generative Model for pLSA

- For each position in d, $n = 1, ..., N_d$
 - Generate the topic for the position as

$$z_n|d\sim Categorical(\boldsymbol{\theta}_d), i.e., p(z_n=k|d)=\theta_{dk}$$
 (Note, 1 trial multinomial)

Generate the word for the position as

$$w_n|z_n\sim Categorical(\boldsymbol{\beta}_{z_n})$$
, i.e., $p(w_n=w|z_n)=\beta_{z_nw}$

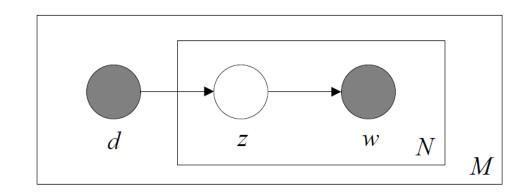
Graphical Model

Generative Model for pLSA

- For each position in d, $n = 1, ..., N_d$
 - Generate the topic for the position as $z_n|d\sim Categorical(\boldsymbol{\theta}_d), i.e., p(z_n=k|d)=\theta_{dk}$ (Note, 1 trial multinomial)
 - Generate the word for the position as

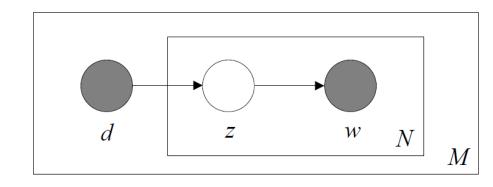
$$w_n|z_n\sim Categorical(\boldsymbol{\beta}_{z_n})$$
, i.e., $p(w_n=w|z_n)=\beta_{z_nw}$





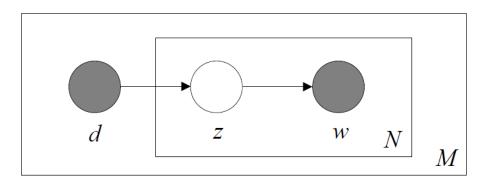
Probability of a word w

 $p(w|d,\theta,\beta)$



Probability of a word w

$$p(w|d,\theta,\beta) = \sum_{k} p(w,z = k|d,\theta,\beta)$$
$$= \sum_{k} p(w|z = k,d,\theta,\beta)p(z = k|d,\theta,\beta) = \sum_{k} \beta_{kw}\theta_{dk}$$



Probability of a word w

$$p(w|d,\theta,\beta) = \sum_{k} p(w,z = k|d,\theta,\beta)$$
$$= \sum_{k} p(w|z = k,d,\theta,\beta)p(z = k|d,\theta,\beta) = \sum_{k} \beta_{kw}\theta_{dk}$$

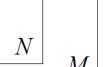
Likelihood of a corpus

$$\prod_{d=1}^{n} P(w_1, \dots, w_{N_d}, d | \theta, \beta, \pi)$$

$$= \prod_{d=1}^{n} P(d) \left\{ \prod_{n=1}^{N_d} \left(\sum_k P(z_n = k | d, \theta_d) P(w_n | \beta_k) \right) \right\}$$

$$= \prod_{d=1}^{n} \pi_d \left\{ \prod_{n=1}^{N_d} \left(\sum_k \theta_{dk} \beta_{kw_n} \right) \right\}$$

 π_d is usually considered as uniform, i.e., 1/M



Re-arrange the Likelihood Function

 Group the same word from different positions together

$$\max log L = \sum_{dw} c(w, d) log \sum_{z} \theta_{dz} \beta_{zw}$$
$$s.t. \sum_{z} \theta_{dz} = 1 \text{ and } \sum_{w} \beta_{zw} = 1$$

Limitations of pLSA

- Not a proper generative model
 - θ_d is treated as a parameter
 - Cannot model new documents

• Solution:

• Make it a proper generative model by adding priors to θ and β

Limitations of pLSA

- Not a proper generative model
 - θ_d is treated as a parameter
 - Cannot model new documents

• Solution:

• Make it a proper generative model by adding priors to θ and β



Topic Model v3: Latent Dirichlet Allocation (LDA)

Review: Dirichlet Distribution

• Dirichlet distribution: $\theta \sim Dirichlet(\alpha)$

• i.e.,
$$p(\boldsymbol{\theta}|\boldsymbol{\alpha}) = \frac{\Gamma(\sum_{k} \alpha_{k})}{\prod_{k} \Gamma(\alpha_{k})} \prod_{k} \theta_{k}^{\alpha_{k}-1}$$
, where $\alpha_{k} > 0$

•
$$\Gamma(\cdot)$$
 is gamma function: $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$
• $\Gamma(z+1) = z\Gamma(z)$

Review: Dirichlet Distribution

• Dirichlet distribution: $\theta \sim Dirichlet(\alpha)$

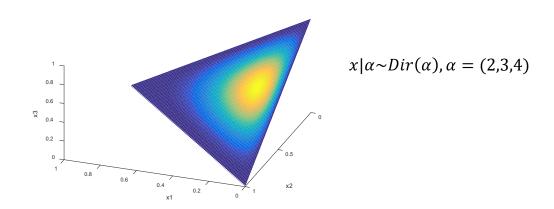
• i.e.,
$$p(\boldsymbol{\theta}|\boldsymbol{\alpha}) = \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} \prod_k \theta_k^{\alpha_k - 1}$$
, where $\alpha_k > 0$

• $\Gamma(\cdot)$ is gamma function: $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$ • $\Gamma(z+1) = z\Gamma(z)$

Simplex view:

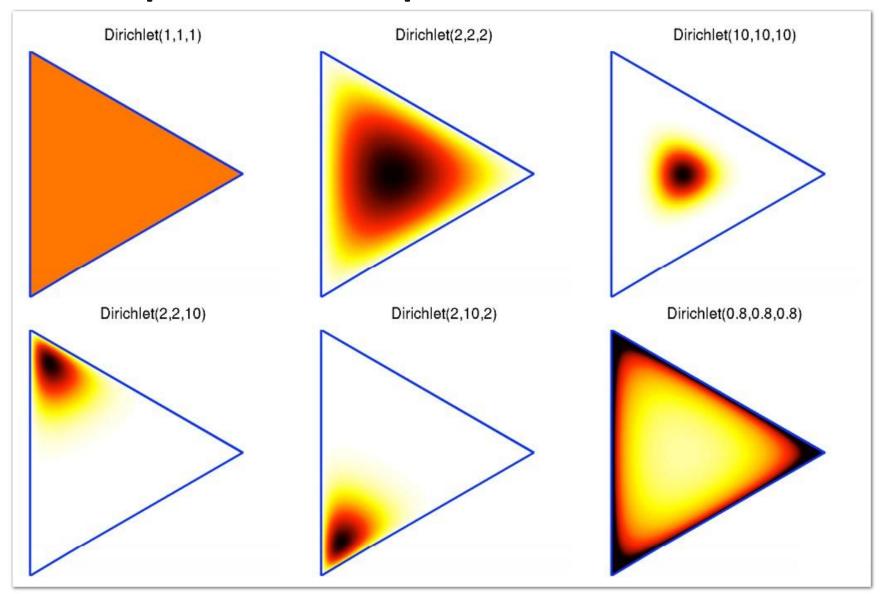
$$\cdot x = x_1(1,0,0) + x_2(0,1,0) + x_3(0,0,1)$$

• Where $0 \le x_1, x_2, x_3 \le 1$ and $x_1 + x_2 + x_3 = 1$

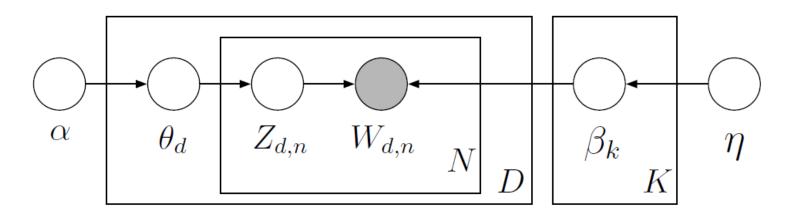


36 41

More Examples in the Simplex View

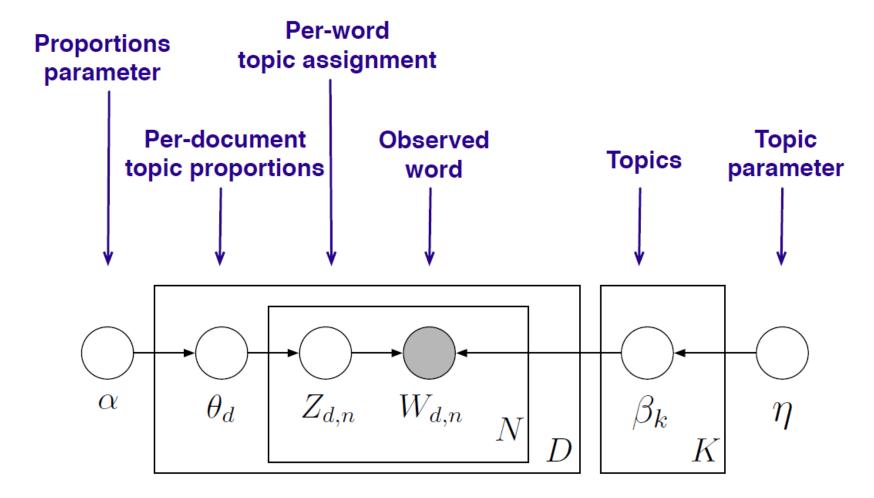


Topic Model v3: Latent Dirichlet Allocation (LDA)



 $\theta_d \sim Dirichlet(\alpha)$: address topic distribution for unseen documents $\beta_k \sim Dirichlet(\eta)$: smoothing over words

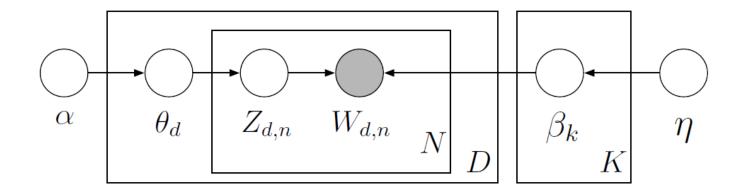
Topic Model v3: Latent Dirichlet Allocation (LDA)

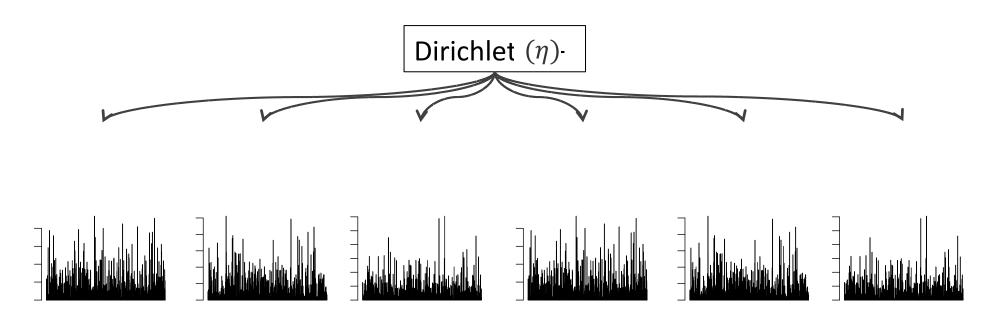


 $\theta_d \sim Dirichlet(\alpha)$: address topic distribution for unseen documents $\beta_k \sim Dirichlet(\eta)$: smoothing over words

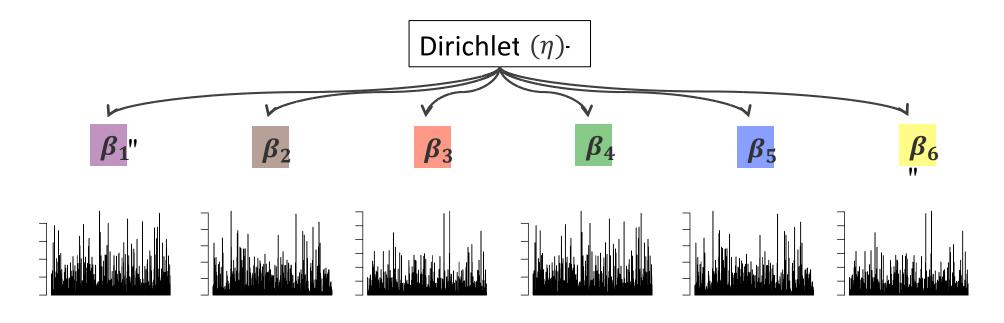
Generative Model for LDA

```
For each topic k \in \{1, \dots, K\}:
\beta_k \sim \operatorname{Dir}(\eta) \qquad [draw \ distribution \ over \ words]
For each document d \in \{1, \dots, D\}
\theta_d \sim \operatorname{Dir}(\alpha) \qquad [draw \ distribution \ over \ topics]
For each word n \in \{1, \dots, N_d\}
z_{d,n} \sim \operatorname{Mult}(1, \theta_d) \qquad [draw \ topic \ assignment]
w_{d,n} \sim \theta_{z_{d,n}} \qquad [draw \ word]
```

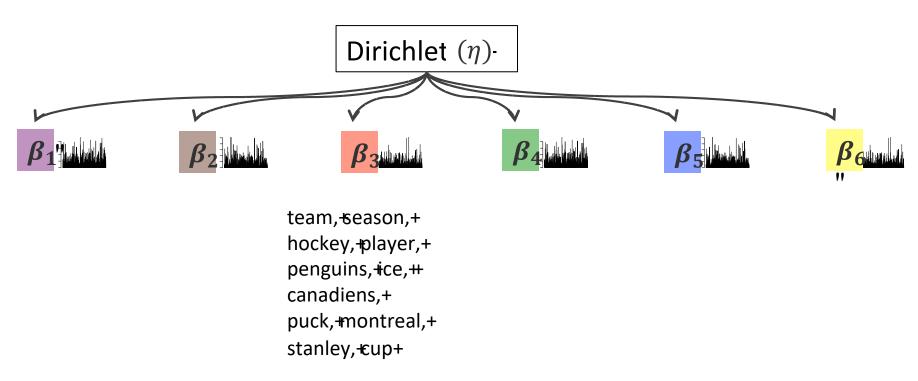




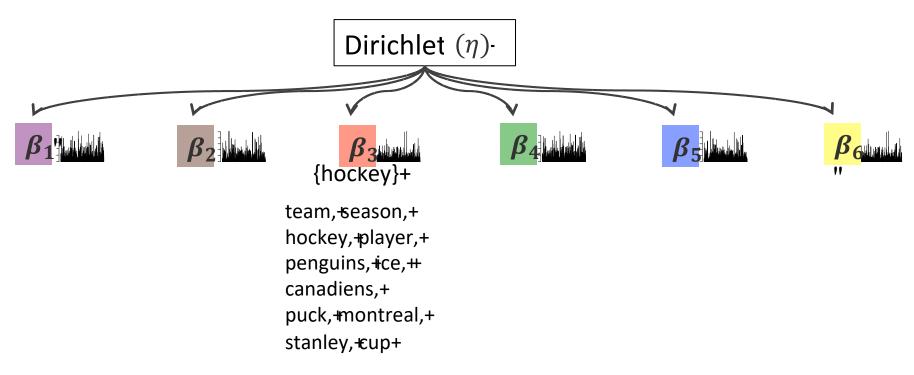
- The'generative&tory&egins'with'bnly'a'Dirichlet& prior&ver'the'topics."
- Each'topic8s'defind 'as'a'Multinomial&distribution' over'the'vocabulary,'parameterized'by β_k '



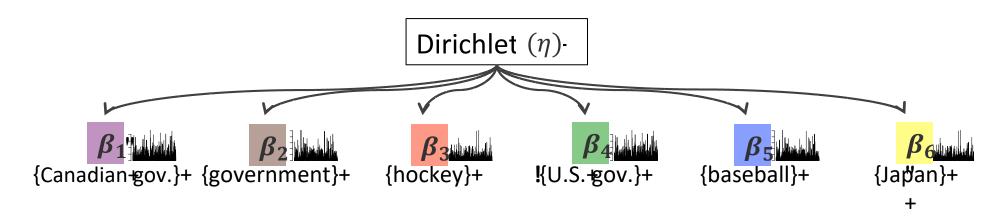
- The'generative&tory&egins'with'bnly'a'Dirichlet& prior&ver'the'topics."
- Each'topic&s'defind 'as'a'Multinomial&distribution'' over'the'vocabulary,'parameterized'by β_k '



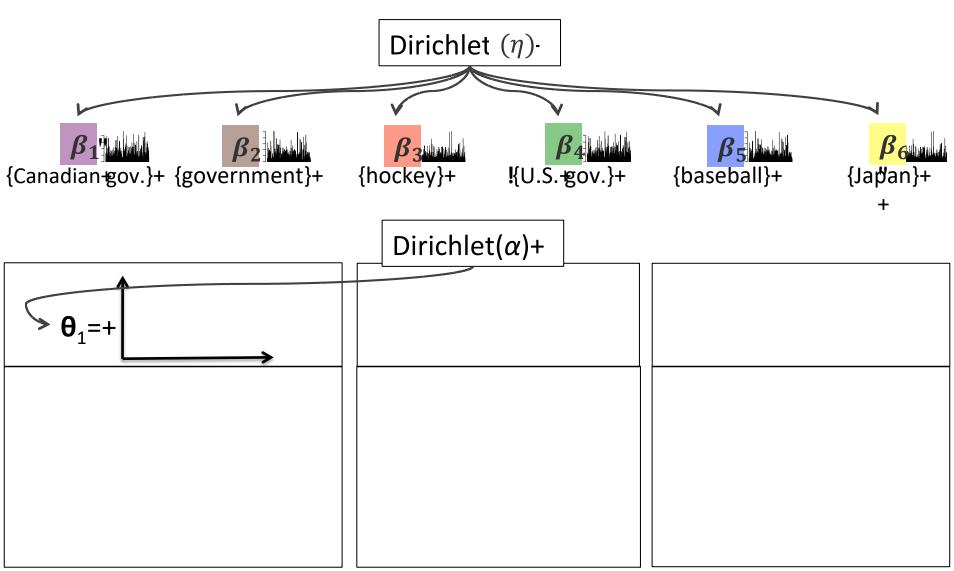
• A'topic'ls'visualized'as'lts'high&robability& words.'''

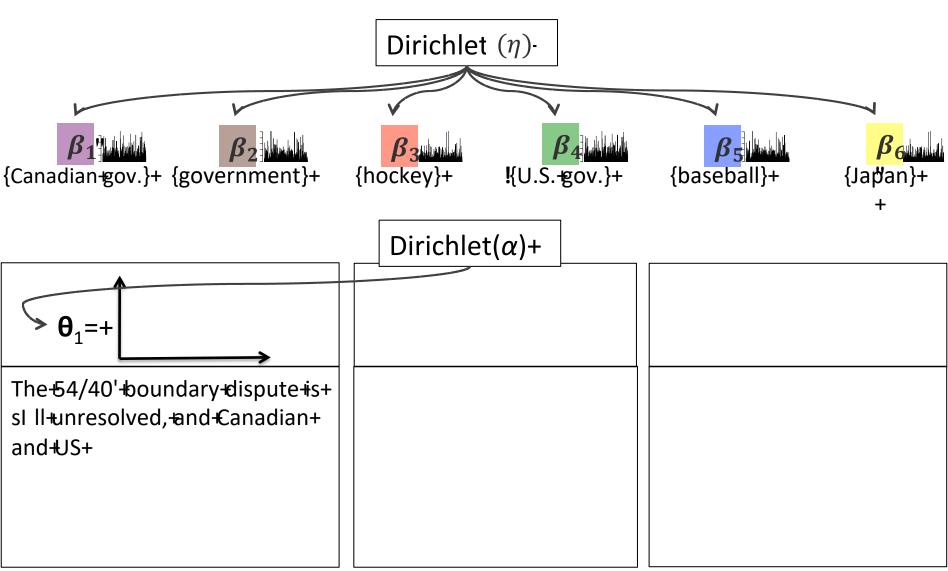


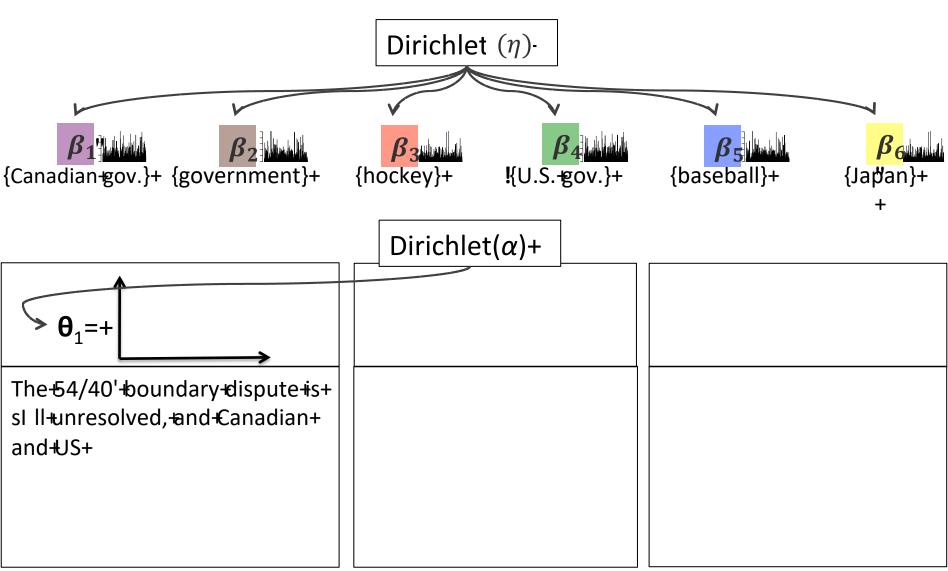
- A'topic'ls'visualized'as'lts'high&robability& words.'"
- A'pedagogical'label&s'used'to'ldentify'the'topic."

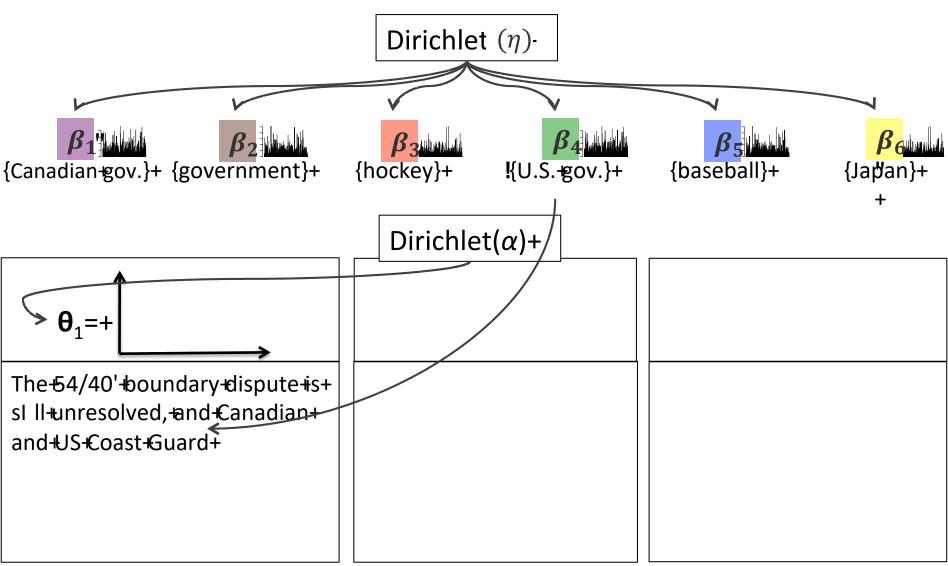


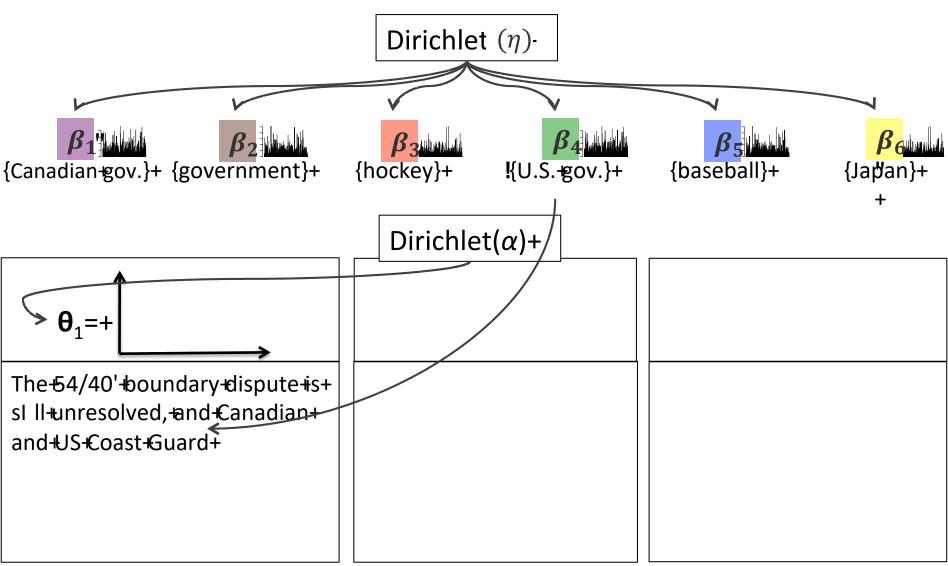
- A'topic'ls'visualized'as'lts'high'probability" words.'"
- A'pedagogical'label&s'used'to'ldentify'the'topic."

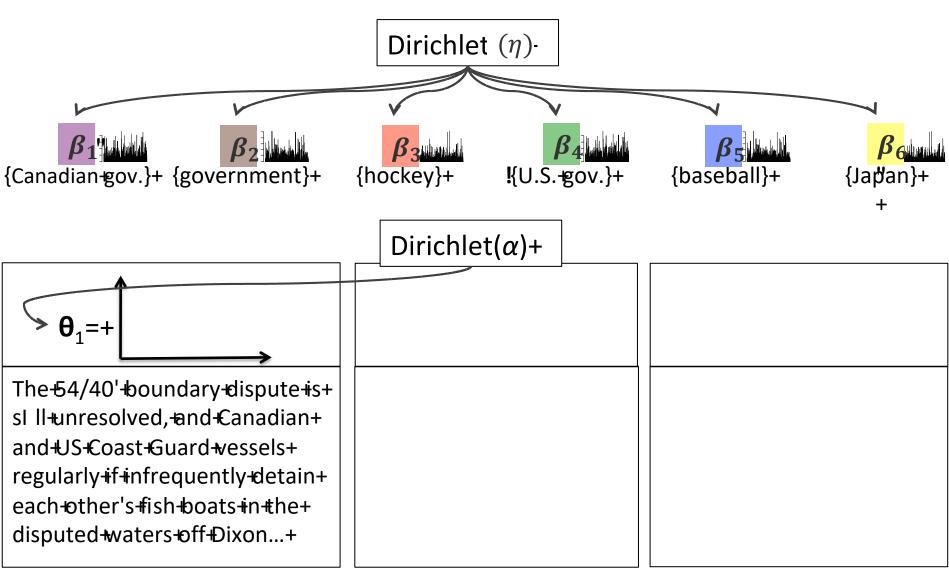


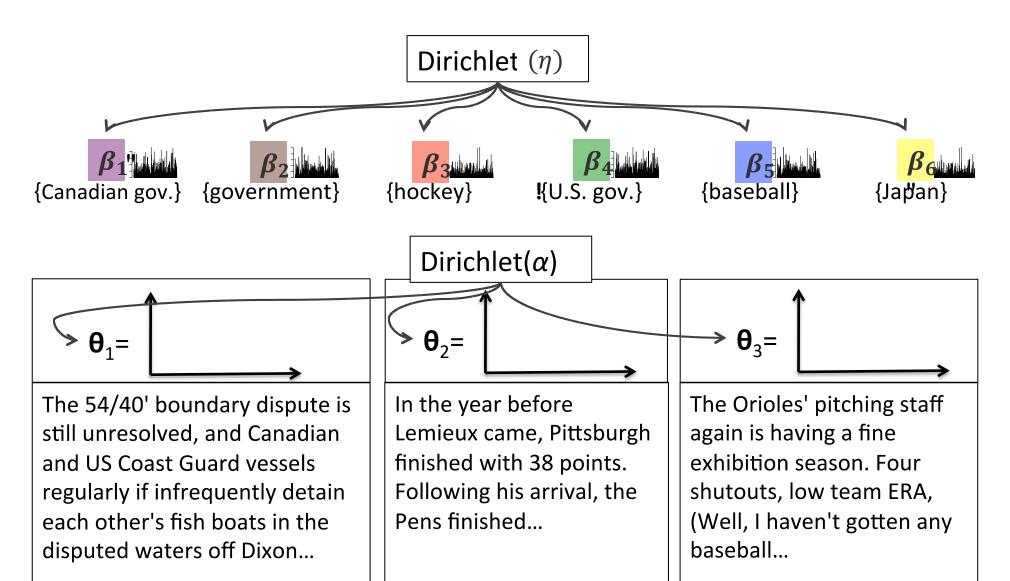


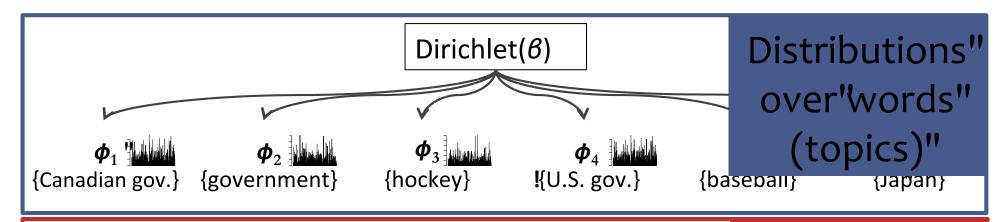


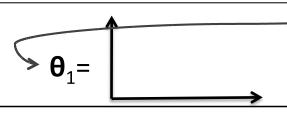












The 54/40' boundary dispute is still unresolved, and Canadian and US Coast Guard vessels regularly if infrequently detain each other's fish boats in the disputed waters off Dixon...

Dirichlet(α)

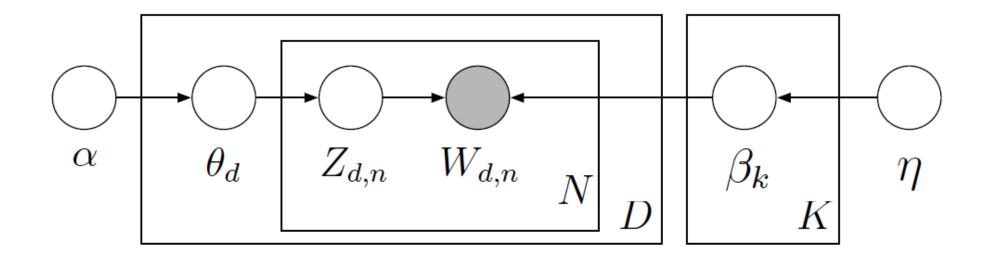
 θ_2 =

In the year before Lemieux came, Pittsburgh finished with 38 points. Following his arrival, the Pens finished...

Distributions" over" topics'(docs)"

The Orioles' pitching staff again is having a fine exhibition season. Four shutouts, low team ERA, (Well, I haven't gotten any baseball...

Joint Distribution for LDA



 Joint distribution of latent variables and documents is:

$$p(\boldsymbol{\beta}_{1:K}, \boldsymbol{z}_{1:D}, \boldsymbol{\theta}_{1:D}, \boldsymbol{w}_{1:D} | \alpha, \eta) =$$

$$\prod\nolimits_{i=1}^{K} p(\beta_{i} | \eta) \prod\nolimits_{d=1}^{D} p(\theta_{d} | \alpha) \left(\prod\nolimits_{n=1}^{N} p(z_{d,n} | \theta_{d}) p(w_{d,n} | \beta_{1:K}, z_{d,n}) \right)$$

Questions?