DSC250: Advanced Data Mining

Topic Models

Zhiting Hu

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UC San Diego
HALICIOĞLU DATA SCIENCE INSTITUTE

In-class paper presentation

Adversarial Examples are not Bugs, they are Features

Andrew Ilyas*

Dimitris Tsipras*

Shibani Santurkar*

Logan Engstrom*

Brandon Tran

Aleksander Mądry

Topic Models

Outline

- Representations of Text and Topics
- Topic Model v1: Multinomial Mixture Model
- Topic Model v2: Probabilistic Latent Semantic Analysis (pLSA)
- Topic Model v3: Latent Dirichlet Allocation (LDA)

Slides adapted from:

- Y. Sun, CS 247: Advanced Data Mining
- M. Gormley, 10-701 Introduction to Machine Learning

Motivation

Suppose you're given a massive corpora and asked to carry out the following tasks

- **Organize** the documents into **thematic categories**
- **Describe** the evolution of those categories **over time**
- Enable a domain expert to **analyze and understand** the content
- Find **relationships** between the categories
- Understand how **authorship** influences the content

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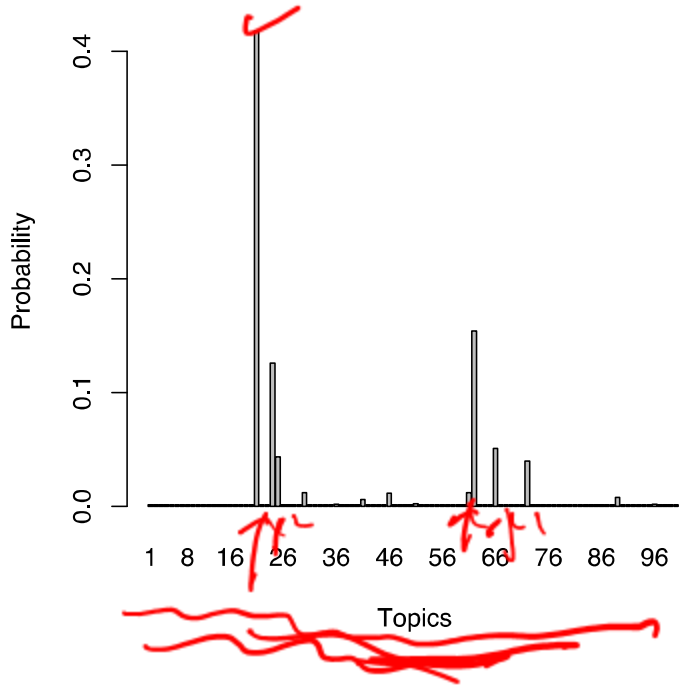
Topic Modeling:

A method of (usually unsupervised) discovery of latent or hidden structure in a corpus

- Applied primarily to text corpora, but **techniques are more general**
- Provides a **modeling toolbox**
- Has prompted the exploration of a variety of new **inference methods** to accommodate **large-scale datasets**

Topic Modeling: Examples



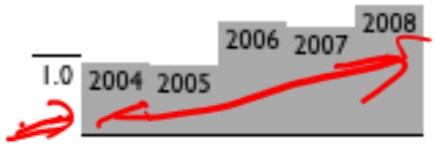
"Genetics"	"Evolution"	"Disease"	"Computers"
human	evolution	disease	computer
genome	evolutionary	host	models
dna	species	bacteria	information
genetic	organisms	diseases	data
genes	life	resistance	computers
sequence	origin	bacterial	system
gene	biology	new	network
molecular	groups	strains	systems
sequencing	phylogenetic	control	model
map	living	infectious	parallel
information	diversity	malaria	methods
genetics	group	parasite	networks
mapping	new	parasites	software
project	two	united	new
sequences	common	tuberculosis	simulations

Figure from (Blei, 2011), shows topics and top words learned automatically from reading 17,000 Science articles

Topic Modeling: Examples

Dirichlet-multinomial regression (DMR) topic model on ICML
(Mimno & McCallum, 2008)

Topic 0 [0.152]

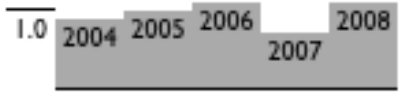


problem, optimization, problems, convex, convex optimization, linear, semidefinite programming, formulation, sets, constraints, proposed, margin, maximum margin, optimization problem, linear programming, programming, procedure, method, cutting plane, solutions

Topic 54 [0.051]

decision trees, trees, tree, decision tree, decision, tree ensemble, junction tree, decision tree learners, leaf nodes, arithmetic circuits, ensembles modts, skewing, ensembles, anytime induction decision trees, trees trees, random forests, objective decision trees, tree learners, trees grove, candidate split

Topic 99 [0.066]



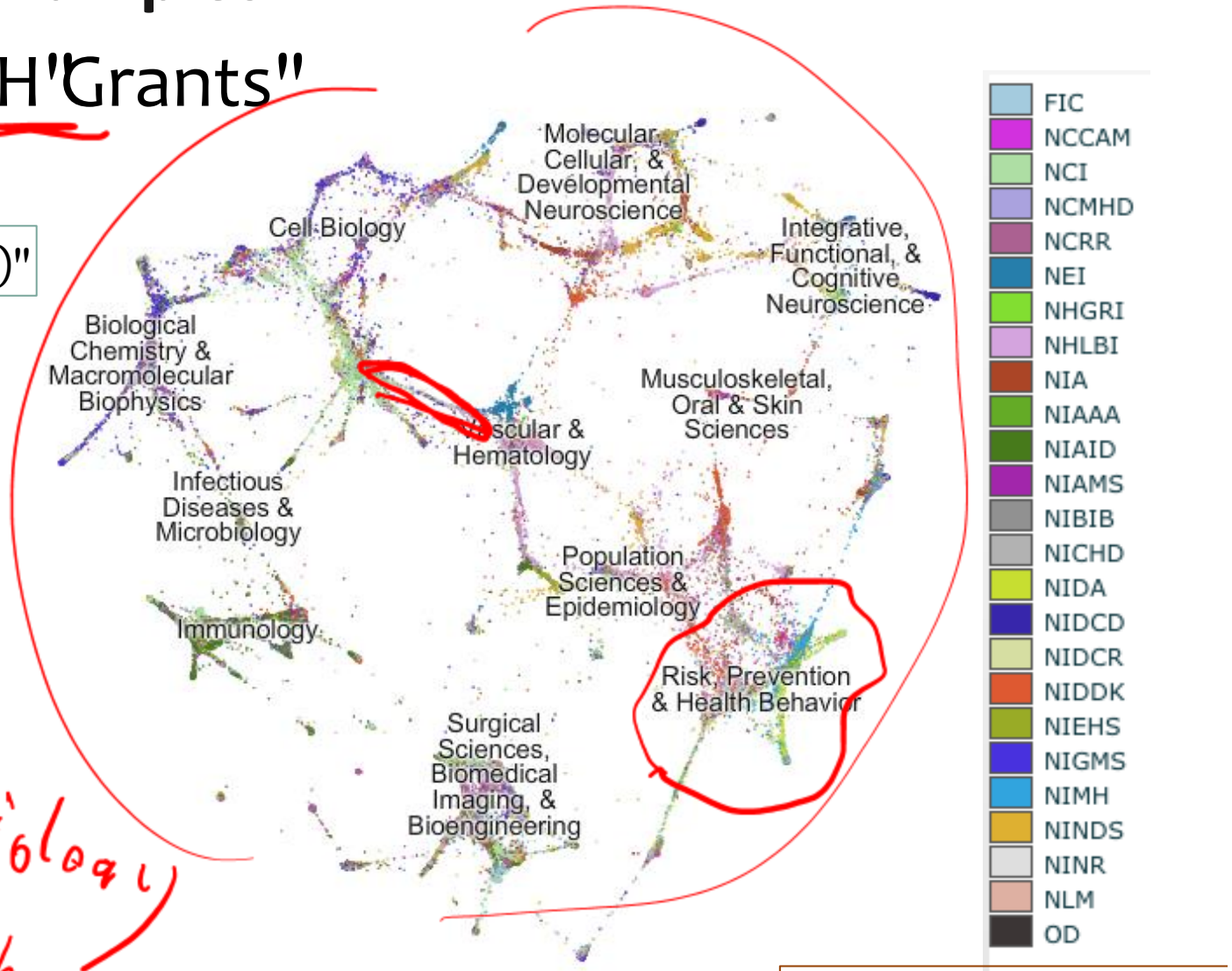
inference, approximate inference, exact inference, markov chain, models, approximate, gibbs sampling, variational, bayesian, variational inference, variational bayesian, approximation, sampling, methods, exact, bayesian inference, dynamic bayesian, process, mcmc, efficient

http://www.cs.umass.edu/~mimno/icml100.html

Topic Modeling: Examples

- Map of NIH Grants

(Talley et al., 2011)

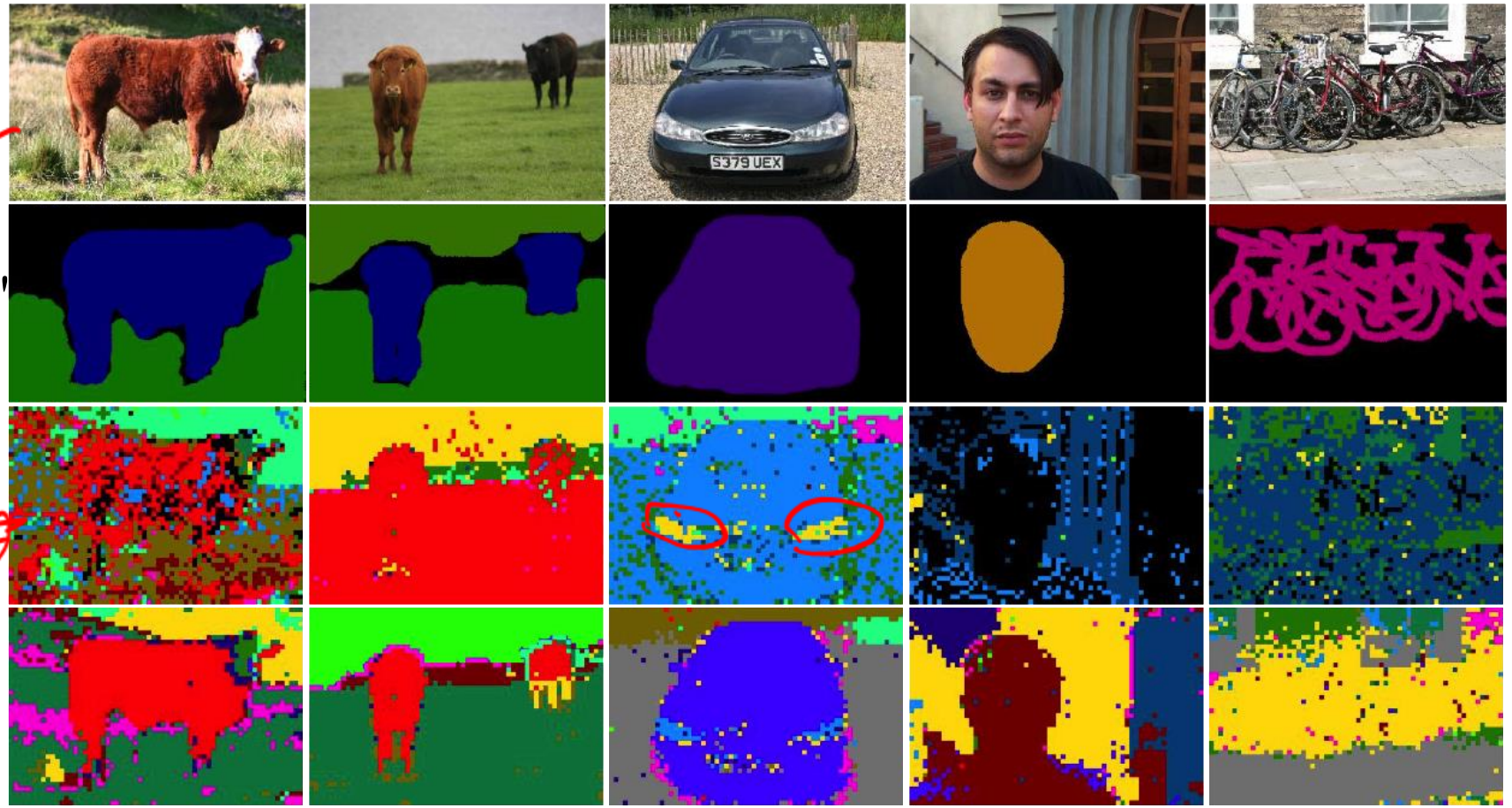


https://app.nihmaps.org/

Other Applications of Topic Models

- Spacial"LDA"

(Wang"&"Grimson,"2007)"



Other Applications of Topic Models

- Word Sense Induction

(Brody & Lapata, 2009)

Senses of *drug* (WSJ)
1. U.S., administration, federal, against, war, dealer
2. patient, people, problem, doctor, company, abuse
3. company, million, sale, maker, stock, inc.
4. administration, food, company, approval, FDA

Senses of *drug* (BNC)
1. patient, treatment, effect, anti-inflammatory
2. alcohol, treatment, patient, therapy, addiction
3. patient, new, find, effect, choice, study
4. test, alcohol, patient, abuse, people, crime
5. trafficking, trafficker, charge, use, problem
6. abuse, against, problem, treatment, alcohol
7. people, wonder, find, prescription, drink, addict
8. company, dealer, police, enforcement, patient

- Selectional Preference

(Ritter et al., 2010)

Topic t	Arg1	Relations which assign highest probability to t	Arg2
18	The residue - The mixture - The reaction mixture - The solution - the mixture - the reaction mixture - the residue - The reaction - the solution - The filtrate - the reaction - The product - The crude product - The pellet - The organic layer - Thereto - This solution - The resulting solution - Next - The organic phase - The resulting mixture - C.)	was treated with, is treated with, was poured into, was extracted with, was purified by, was diluted with, was filtered through, is disolved in, is washed with	EtOAc - CH2Cl2 - H2O - CH.sub.2Cl.sub.2 - H.sub.2O - water - MeOH - NaHCO3 - Et2O - NHCl - CHCl.sub.3 - NHCl - dropwise - CH2Cl.sub.2 - Celite - Et.sub.2O - Cl.sub.2 - NaOH - AcOEt - CH2C12 - the mixture - saturated NaHCO3 - SiO2 - H2O - N hydrochloric acid - NHCl - preparative HPLC - to0 C

Text Data

- Word/term
- Document
  - A sequence of words
- Corpus
  - A collection of documents

Represent a Document

- Most common way: Bag-of-Words
  - Ignore the order of words
  - keep the count

c1: *Human* machine *interface* for Lab ABC *computer* applications
c2: A *survey* of *user* opinion of *computer system response time*
c3: The *EPS user interface* management *system*
c4: *System* and *human system* engineering testing of *EPS*
c5: Relation of *user*-perceived *response time* to error measurement

m1: The generation of random, binary, unordered *trees*
m2: The intersection *graph* of paths in *trees*
m3: *Graph minors* IV: Widths of *trees* and well-quasi-ordering
m4: *Graph minors*: A *survey*

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

Vector space model

Represent a Document

- Represent the doc as a vector where each entry corresponds to a different word and the number at that entry corresponds to how many times that word was present in the document (or some function of it)
  - Number of words is huge
  - Select and use a smaller set of words that are of interest
  - E.g. uninteresting words: 'and', 'the' 'at', 'is', etc. These are called stop-words
  - Stemming: remove endings. E.g. 'learn', 'learning', 'learnable', 'learned' could be substituted by the single stem 'learn'
  - Other simplifications can also be invented and used
  - The set of different remaining words is called dictionary or vocabulary. Fix an ordering of the terms in the dictionary so that you can operate them by their index.
  - Can be extended to bi-gram, tri-gram, or so

Limitations of Bag-of-Words

- Dimensionality
  - High dimensionality
- Sparseness
  - Most of the entries are zero
- Shallow representation
  - The vector representation does not capture semantic relations between words

Ex: **"Tom loves Kate."**

Represent a Topic

- A topic is represented by a word distribution
- Relate to an issue

universe	0.0439
galaxies	0.0375
clusters	0.0279
matter	0.0233
galaxy	0.0232
cluster	0.0214
cosmic	0.0137
dark	0.0131
light	0.0109
density	0.01

drug	0.0672
patients	0.0493
drugs	0.0444
clinical	0.0346
treatment	0.028
trials	0.0277
therapy	0.0213
trial	0.0164
disease	0.0157
medical	0.00997

cells	0.0675
stem	0.0478
human	0.0421
cell	0.0309
gene	0.025
tissue	0.0185
cloning	0.0169
transfer	0.0155
blood	0.0113
embryos	0.0111

sequence	0.0818
sequences	0.0493
genome	0.033
dna	0.0257
sequencing	0.0172
map	0.0123
genes	0.0122
chromosome	0.0119
regions	0.0119
human	0.0111

years	0.156
million	0.0556
ago	0.045
time	0.0317
age	0.0243
year	0.024
record	0.0238
early	0.0233
billion	0.0177
history	0.0148

bacteria	0.0983
bacterial	0.0561
resistance	0.0431
coli	0.0381
strains	0.025
microbiol	0.0214
microbial	0.0196
strain	0.0165
salmonella	0.0163
resistant	0.0145

male	0.0558
females	0.0541
female	0.0529
males	0.0477
sex	0.0339
reproductive	0.0172
offspring	0.0168
sexual	0.0166
reproduction	0.0143
eggs	0.0138

theory	0.0811
physics	0.0782
physicists	0.0146
einstein	0.0142
university	0.013
gravity	0.013
black	0.0127
theories	0.01
aps	0.00987
matter	0.00954

immune	0.0909
response	0.0375
system	0.0358
responses	0.0322
antigen	0.0263
antigens	0.0184
immunity	0.0176
immunology	0.0145
antibody	0.014
autoimmune	0.0128

stars	0.0524
star	0.0458
astrophys	0.0237
mass	0.021
disk	0.0173
black	0.0161
gas	0.0149
stellar	0.0127
astron	0.0125
hole	0.00824

Topic Models

- Topic modeling
  - Get topics automatically from a corpus
  - Assign documents to topics automatically
- Most frequently used topic models
  - pLSA
  - LDA

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
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ACTOR	NEW	SAYS	BENNETT
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YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
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Notations

- Word, document, topic
  - w, d, z
- Word count in document:
  - $c(w, d)$: number of times word w occurs in document d
  - or x_{dn}: number of times the nth word in the vocabulary occurs in document d
- Word distribution for each topic (β_z)
  - β_{zw}: $p(w|z)$

Recap: Multinomial distribution

- Multinomial distribution
  - Discrete random variable $\boldsymbol{x}$ that takes one of M values $\{1, \dots, M\}$
  - $p(\boldsymbol{x} = i) = \pi_i, \quad \sum_i \pi_i = 1$
  - Out of n independent trials, let k_i be the number of times $\boldsymbol{x} = i$ was observed
  - The probability of observing a vector of occurrences $\boldsymbol{k} = [k_1, \dots, k_M]$ is given by the *multinomial distribution* parametrized by $\boldsymbol{\pi}$

$$p(\mathbf{k}|\boldsymbol{\pi}, \mathrm{n}) = p(\mathrm{k}_1, \dots, \mathrm{k}_\mathrm{m}|\pi_1, \dots, \pi_\mathrm{m}, \mathrm{n}) = \frac{\mathrm{n}!}{\mathrm{k}_1!\mathrm{k}_2! \dots \mathrm{k}_\mathrm{m}!} \prod_{\mathrm{i}=1} \pi_\mathrm{i}^{\mathrm{k}_\mathrm{i}}$$

  - E.g., describing a text document by the frequency of occurrence of every distinct word
  - For $n = 1$, a.k.a. categorical distribution
    - $p(\boldsymbol{x} = i \mid \boldsymbol{\pi}) = \pi_i$
    - In $\boldsymbol{k} = [k_1, \dots, k_M]$: $k_i = 1$, and $k_j = 0$ for all $j \neq i \quad \rightarrow \quad a.k.a.$, one-hot representation of i

Topic Model v1: Multinomial Mixture Model

- For documents with bag-of-words representation
  - $\boldsymbol{x}_d = (x_{d1}, x_{d2}, \ldots, x_{dN})$, x_{dn} is the number of words for nth word in the vocabulary
- Generative model

Topic Model v1: Multinomial Mixture Model

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Formulating the statistical relationship between words, documents and latent topics as a generative process describing how documents are created

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- Generative model
  - For each document
    - Sample its cluster label $z \sim Categorical(\boldsymbol{\pi})$
      - $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_K)$, π_k is the proportion of jth cluster
      - $p(z = k) = \pi_k$
    - Sample its word vector $\boldsymbol{x}_d \sim multinomial(\boldsymbol{\beta}_z)$
      - $\boldsymbol{\beta}_z = (\beta_{z1}, \beta_{z2}, \dots, \beta_{zN})$, β_{zn} is the parameter associate with nth word in the vocabulary
      - $p(\boldsymbol{x}_d | z = k) = \frac{(\sum_n x_{dn})!}{\prod_n x_{dn}!} \prod_n \beta_{kn}^{x_{dn}} \propto \prod_n \beta_{kn}^{x_{dn}}$

N: vocabulary size
n: length of d_{0i}

Topic Model v1: Multinomial Mixture Model

Graphical Model

- *Plates indicate replicated variables.*
- *Shaded nodes are observed; unshaded nodes are hidden.*

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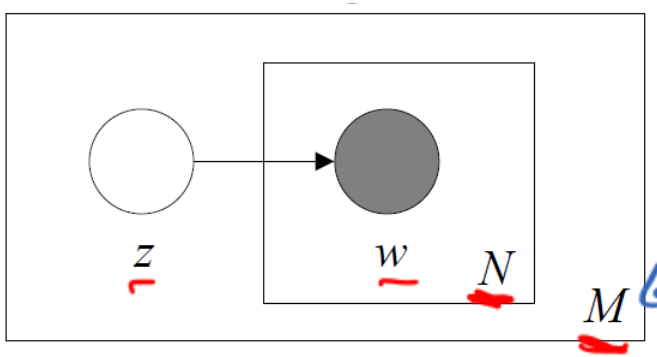

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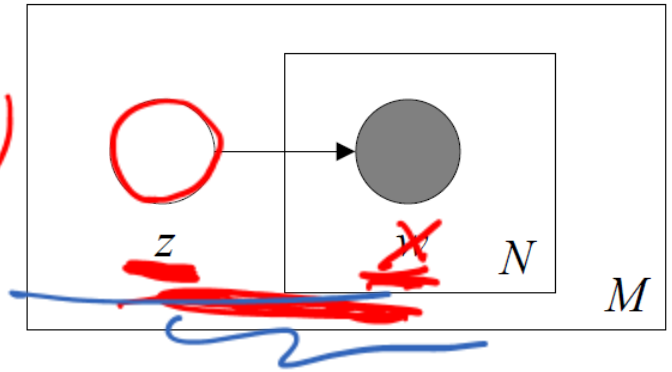
Likelihood Function



$$\prod_d p(\vec{x}_d) = \prod_d \sum_{k=1}^{K} p(\vec{x}_d, z=k)$$

$$= \prod_d \sum_{k=1}^{K} p(z=k)\, p(\vec{x}_d \mid z=k)$$

$$= \prod_d \sum_{k=1}^{K} \pi_k \cdot \frac{(\sum_n x_{dn})!}{\prod_n x_{dn}!} \cdot \beta_{kn}^{x_{dn}}$$

π, β ← model parameters

$$\max_{\pi,\beta} \log \prod_d p(\vec{x}_d)$$

Likelihood Function

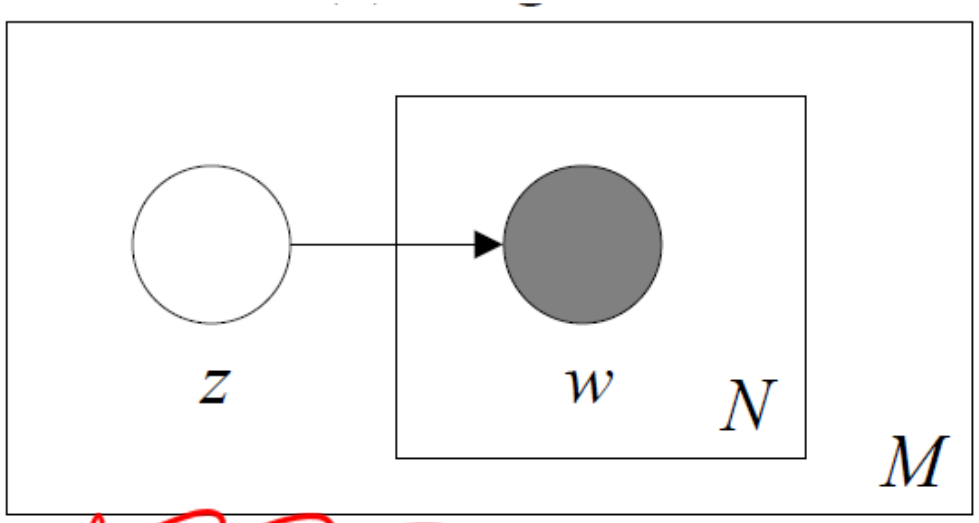
$$L = \prod_d p(\boldsymbol{x}_d) = \prod_d \sum_k p(\boldsymbol{x}_d, z = k)$$

$$= \prod_d \sum_k p(\boldsymbol{x}_d | z = k) p(z = k)$$

$$= \prod_d \frac{(\sum_n x_{dn})!}{\prod_n x_{dn}!} \sum_k p(z = k) \prod_n \beta_{kn}^{x_{dn}}$$

Limitations of Multinomial Mixture Model

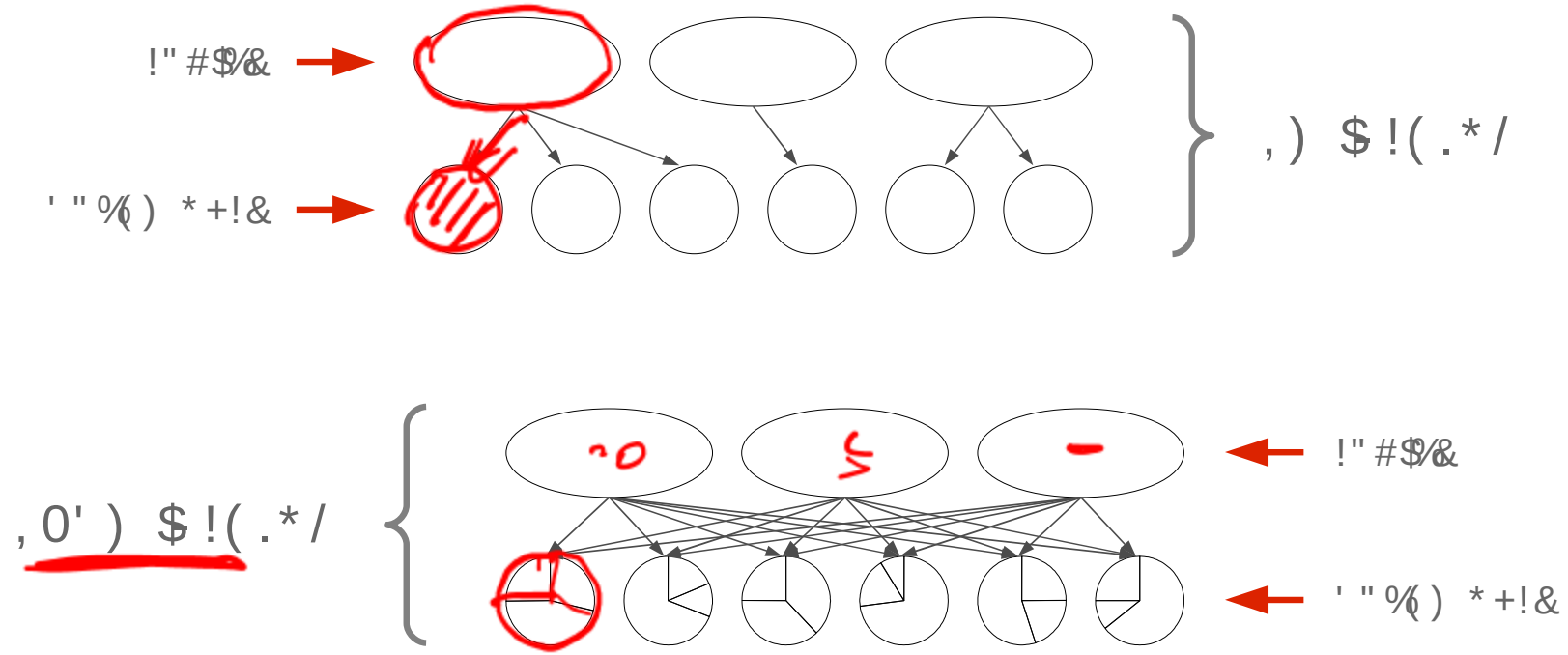
- All the words in the same documents are sampled from the same topic



- In practice, people switch topics during their writing

Limitations of Multinomial Mixture Model

Mixture vs. Admixture



Diagrams from Wallach, JHU 2011, slides

Topic Model v2: Probabilistic Latent Semantic Analysis (pLSA)

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
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ACTOR	NEW	SAYS	BENNETT
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Generative Model for pLSA

- For each position in d, $n = 1, \ldots, N_d$
  - Generate the topic for the position as

$z_n|d \sim Categorical(\boldsymbol{\theta}_d), i.e., p(z_n = k|d) = \theta_{dk}$

(Note, 1 trial multinomial)

  - Generate the word for the position as

$w_n|z_n \sim Categorical(\boldsymbol{\beta}_{z_n}), i.e., p(w_n = w|z_n) = \beta_{z_n w}$

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Graphical Model

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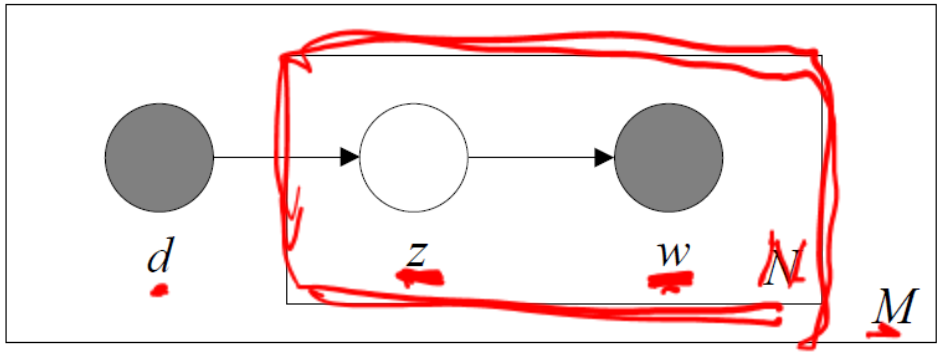
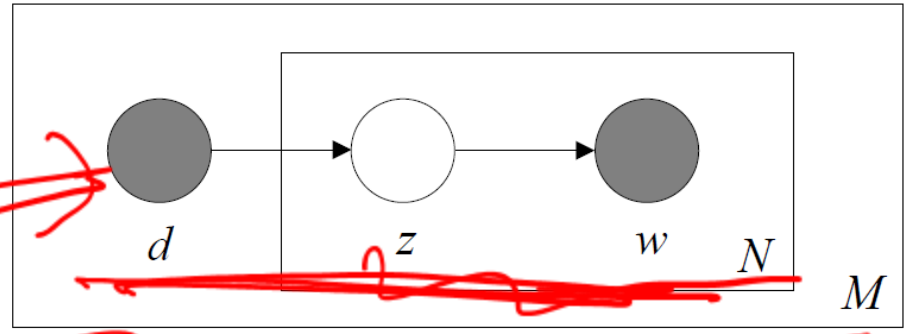


Likelihood Function

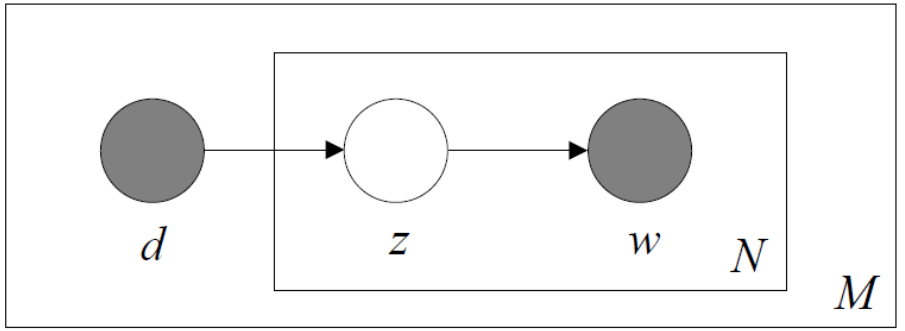
- Probability of a word w

$$p(w|d,\theta,\beta) = \sum_{z} p(w,z|d,\theta,\beta)$$

$$p(z|d)\,p(w|z)$$



Likelihood Function



- Probability of a word w

$$p(w|d,\theta,\beta) = \sum_k p(w, z = k|d,\theta,\beta)$$

$$= \sum_k p(w|z = k, d, \theta, \beta)p(z = k|d,\theta,\beta) = \sum_k \beta_{kw}\theta_{dk}$$

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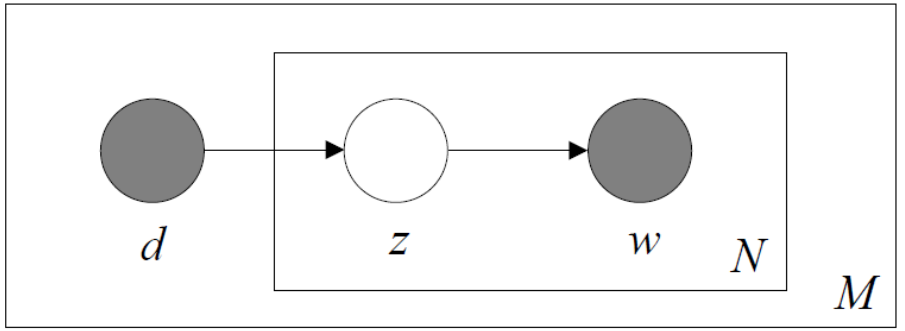

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- Likelihood of a corpus

$$\prod_{d=1} P(w_1,\cdots,w_{N_d},d|\theta,\beta,\pi)$$

$$= \prod_{d=1} P(d)\left\{\prod_{n=1}^{N_d}\left(\sum_k P(z_n=k|d,\theta_d)P(w_n|\beta_k)\right)\right\}$$

$$= \prod_{d=1} \pi_d\left\{\prod_{n=1}^{N_d}\left(\sum_k \theta_{dk}\beta_{kw_n}\right)\right\}$$

π_d *is usually considered as uniform*, i.e., 1/M

Re-arrange the Likelihood Function

- Group the same word from different positions together

$$\max logL = \sum_{dw} c(w,d) log \sum_{z} \theta_{dz}\, \beta_{zw}$$

$$s.t. \sum_{z} \theta_{dz} = 1 \; and \sum_{w} \beta_{zw} = 1$$

Limitations of pLSA

- Not a proper generative model
  - $\boldsymbol{\theta}_d$ is treated as a parameter
  - Cannot model new documents

- Solution:
  - Make it a proper generative model by adding priors to θ and β

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Topic Model v3: Latent Dirichlet Allocation (LDA)

Review: Dirichlet Distribution

- Dirichlet distribution: $\boldsymbol{\theta} \sim Dirichlet(\boldsymbol{\alpha})$
  - $i.e., p(\boldsymbol{\theta}|\boldsymbol{\alpha}) = \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} \prod_k \theta_k^{\alpha_k - 1}$, where $\alpha_k > 0$
    - $\Gamma(\cdot)$ $is\ gamma\ function$: $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$
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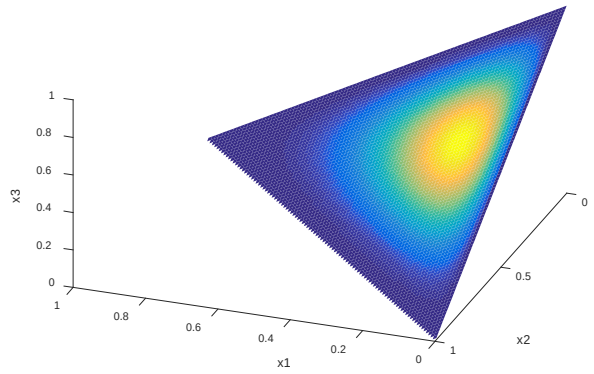
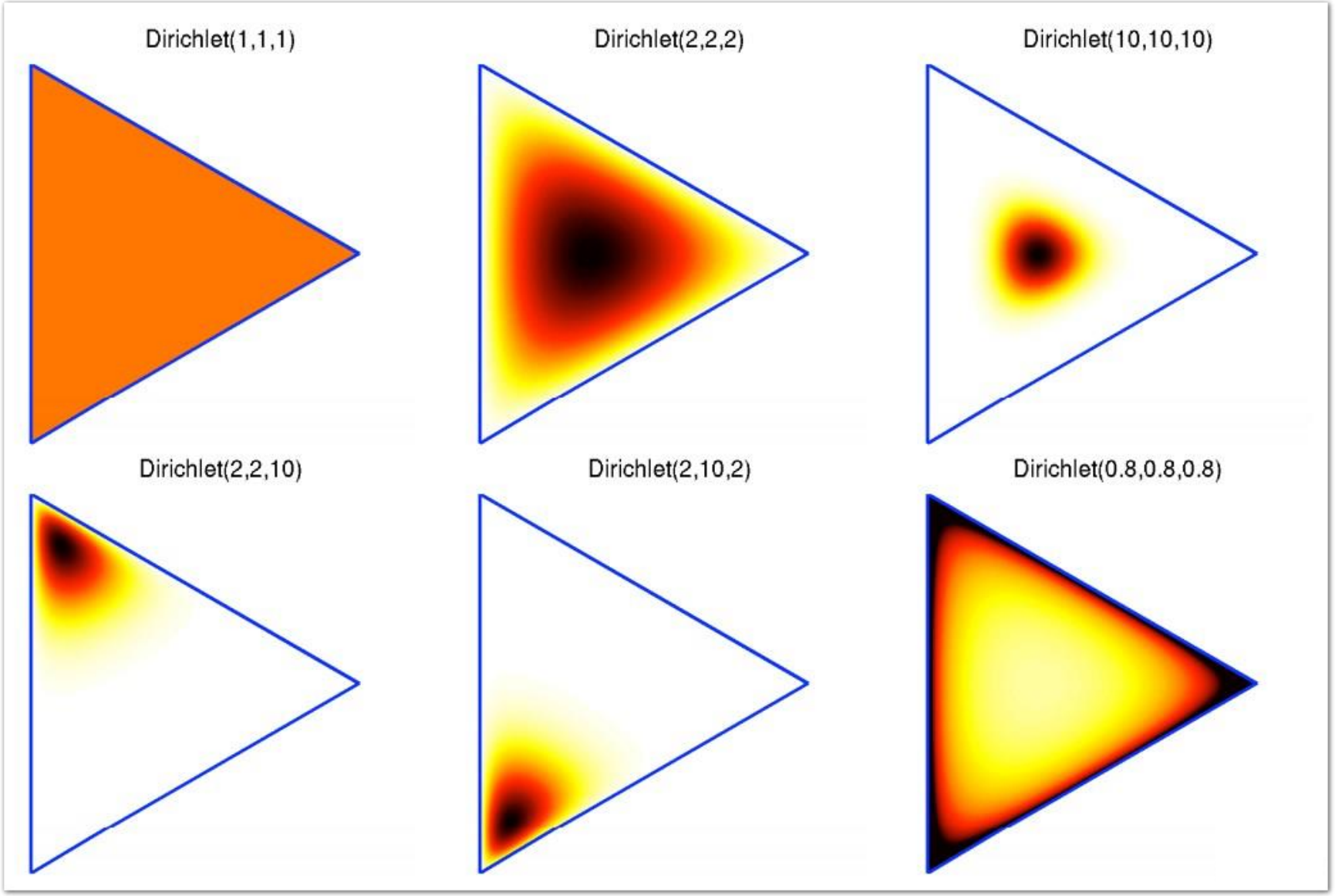
Simplex view:

- $x = x_1(1,0,0) + x_2(0,1,0) + x_3(0,0,1)$
  - Where $0 \leq x_1, x_2, x_3 \leq 1$ *and* $x_1 + x_2 + x_3 = 1$

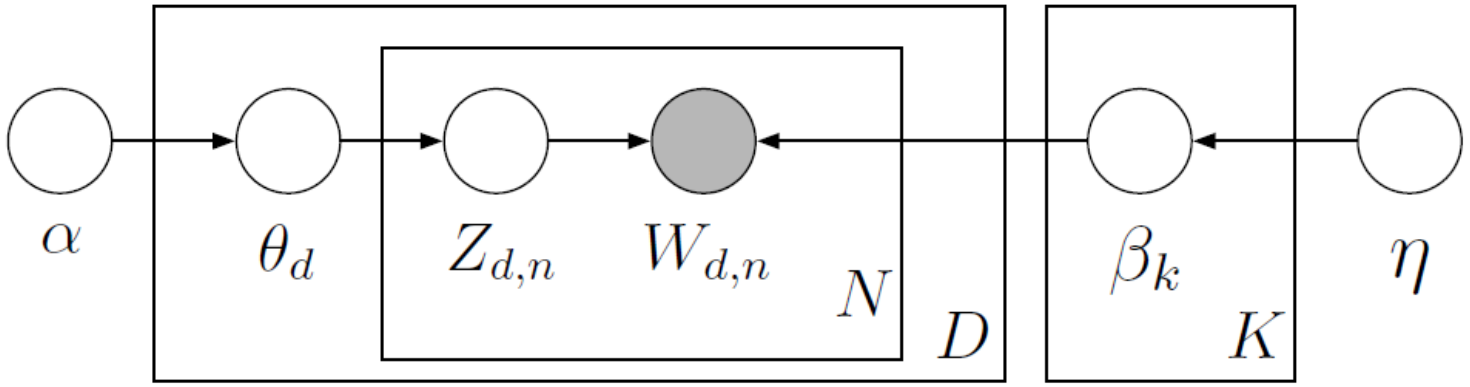


$x|\alpha \sim Dir(\alpha), \alpha = (2,3,4)$

More Examples in the Simplex View



Topic Model v3: Latent Dirichlet Allocation (LDA)



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$\boldsymbol{\beta_k \sim Dirichlet(\eta)}$: smoothing over words

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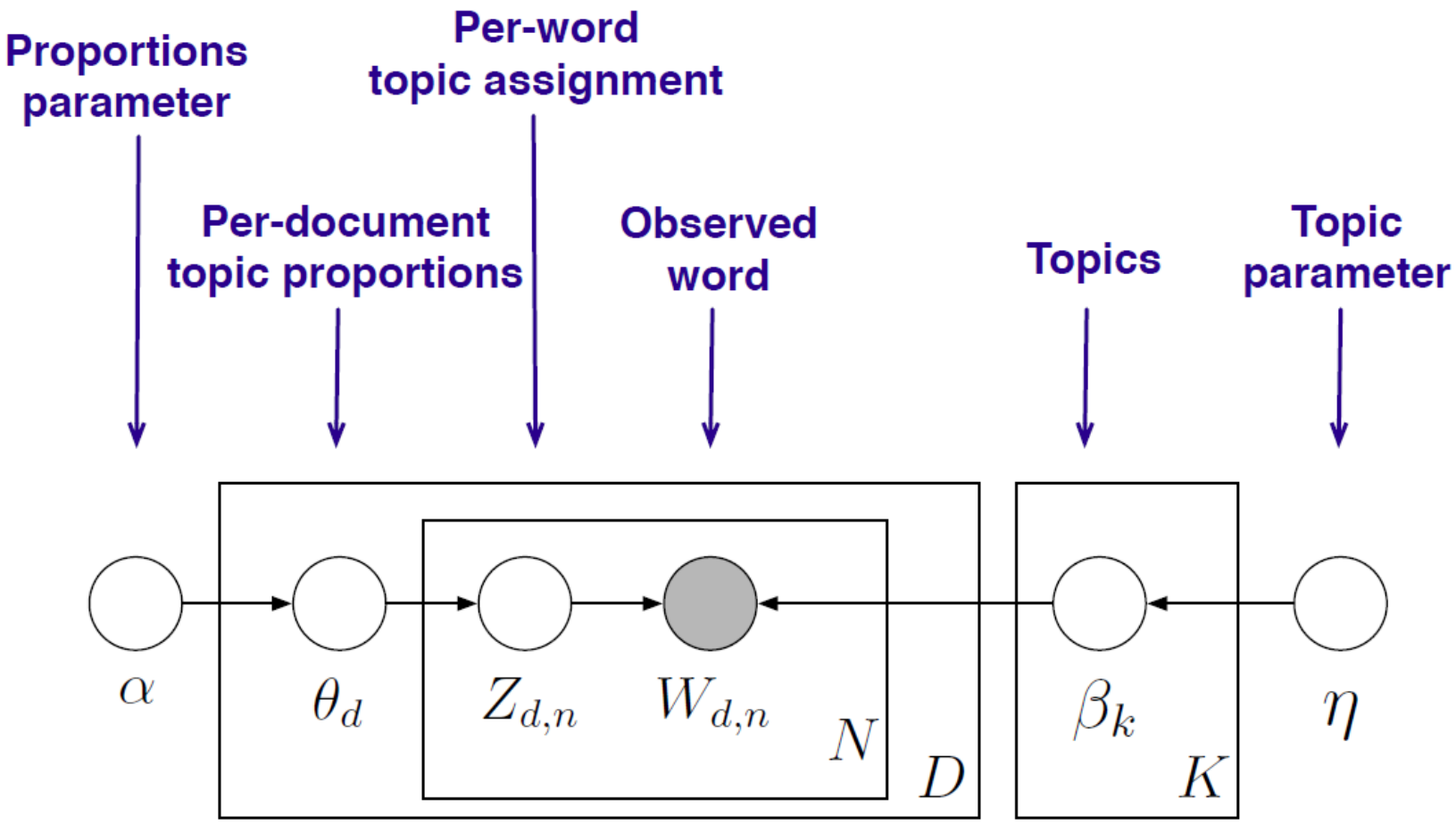

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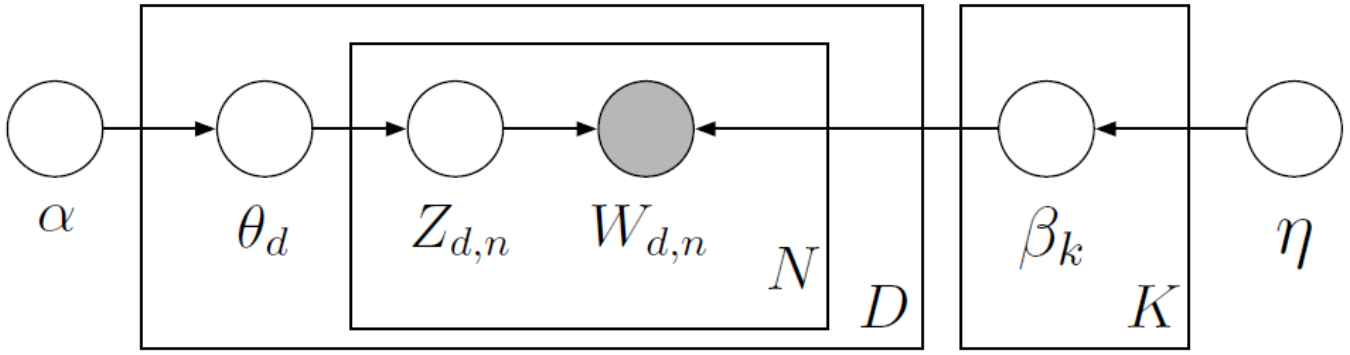
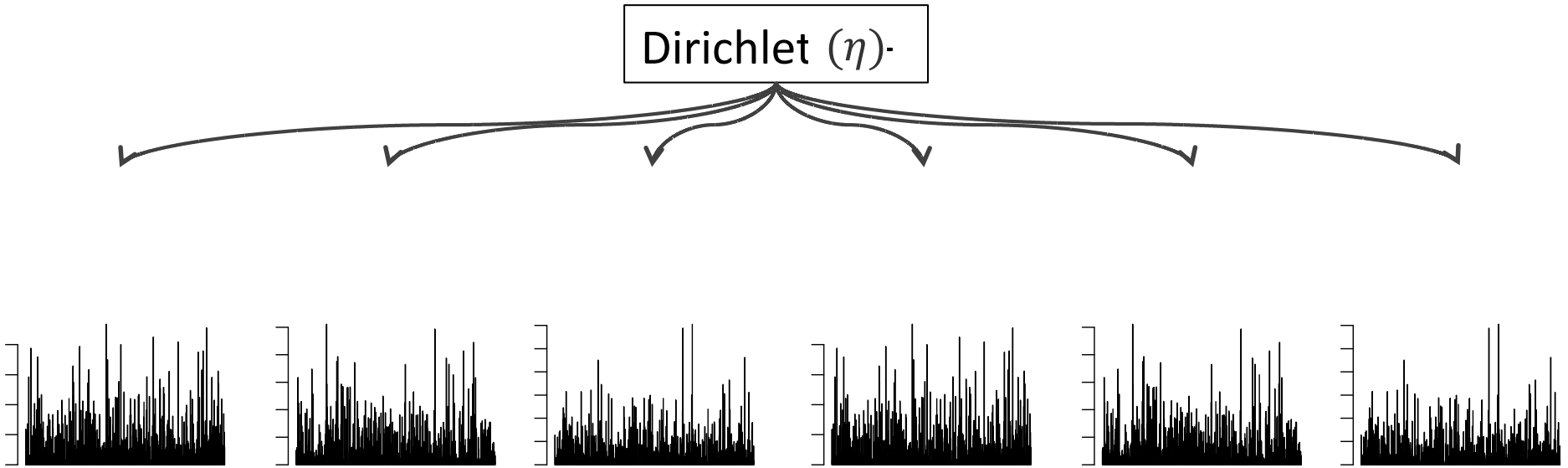
Generative Model for LDA

For each topic $k \in \{1, \ldots, K\}$:
  $\beta_k \sim \text{Dir}(\eta)$ [*draw distribution over words*]
For each document $d \in \{1, \ldots, D\}$
  $\boldsymbol{\theta}_d \sim \text{Dir}(\boldsymbol{\alpha})$ [*draw distribution over topics*]
  For each word $n \in \{1, \ldots, N_d\}$
    $z_{d,n} \sim \text{Mult}(1, \boldsymbol{\theta}_d)$ [*draw topic assignment*]
    $w_{d,n} \sim \theta_{z_{d,n}}$ [*draw word*]



(Blei, Ng, & Jordan, 2003)

LDA for Topic Modeling



- The **generative story** begins with only a **Dirichlet prior** over the topics.
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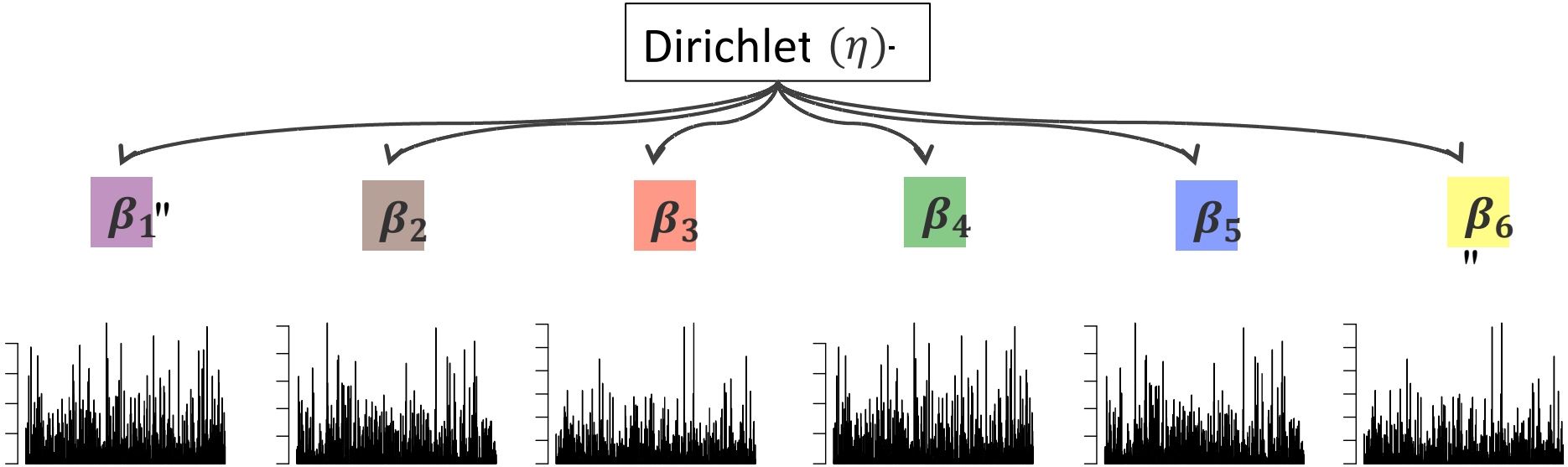

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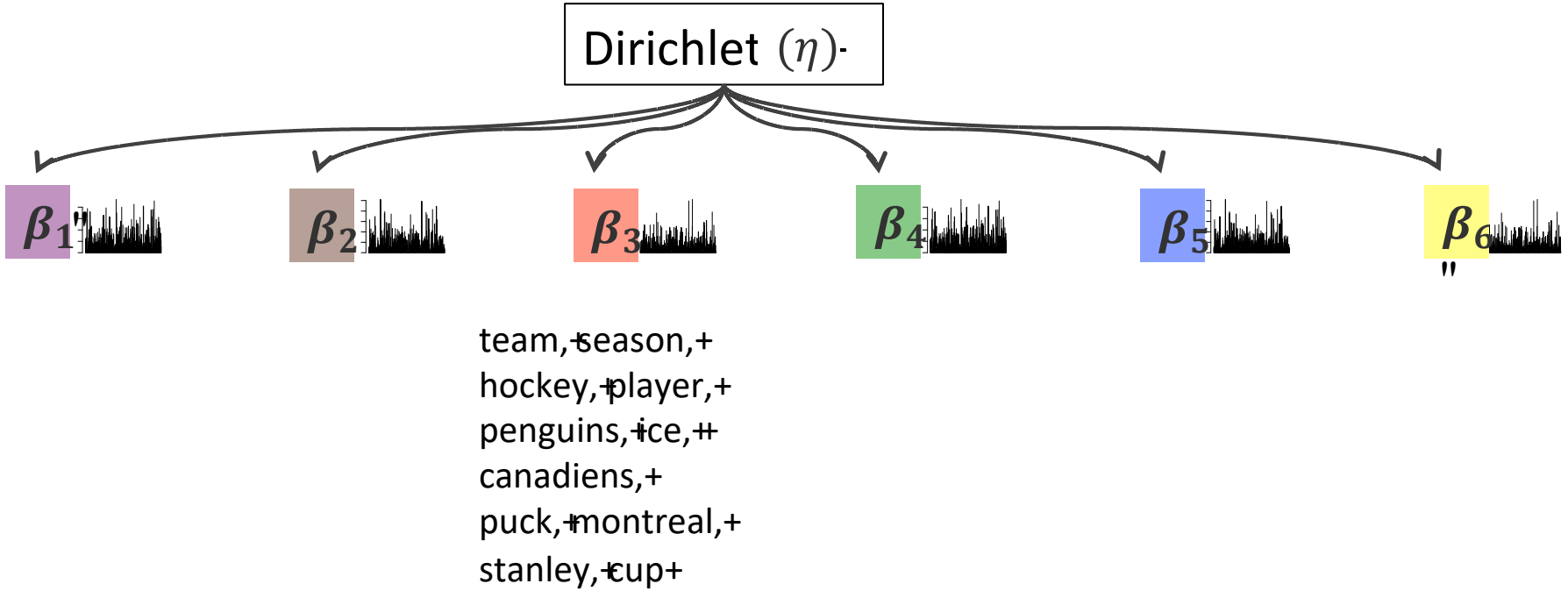

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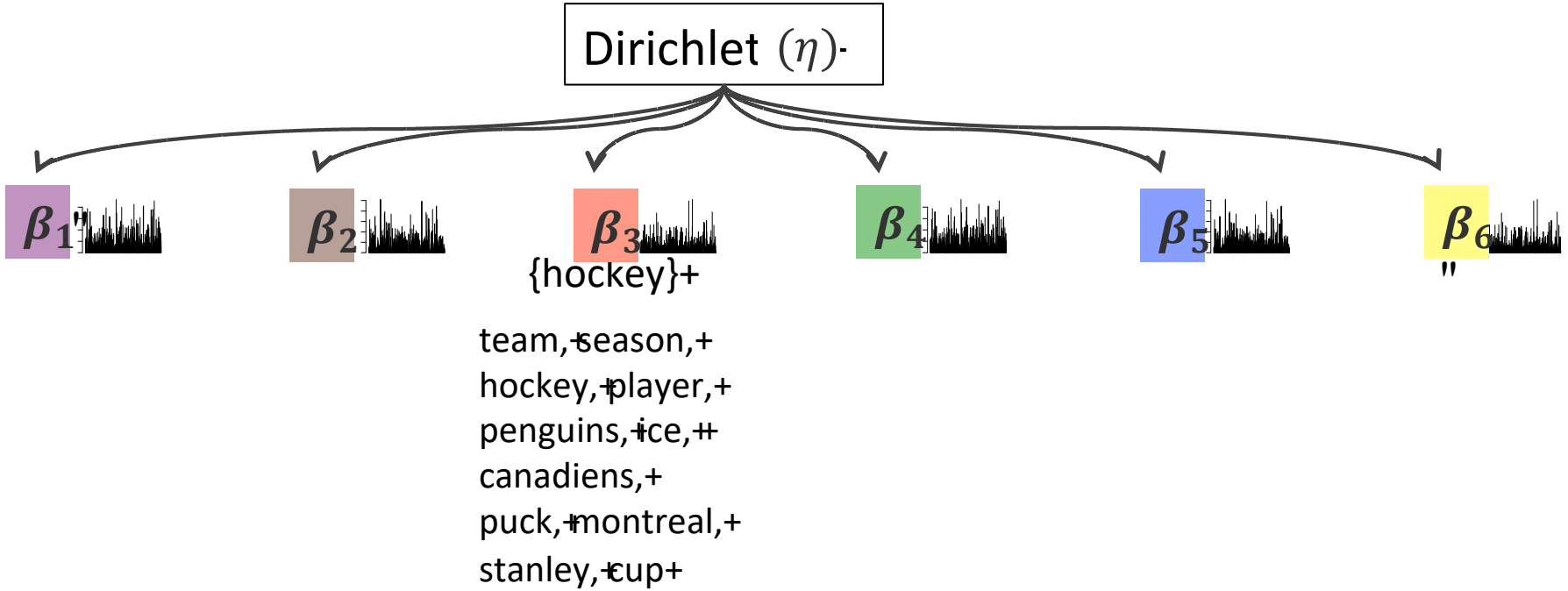

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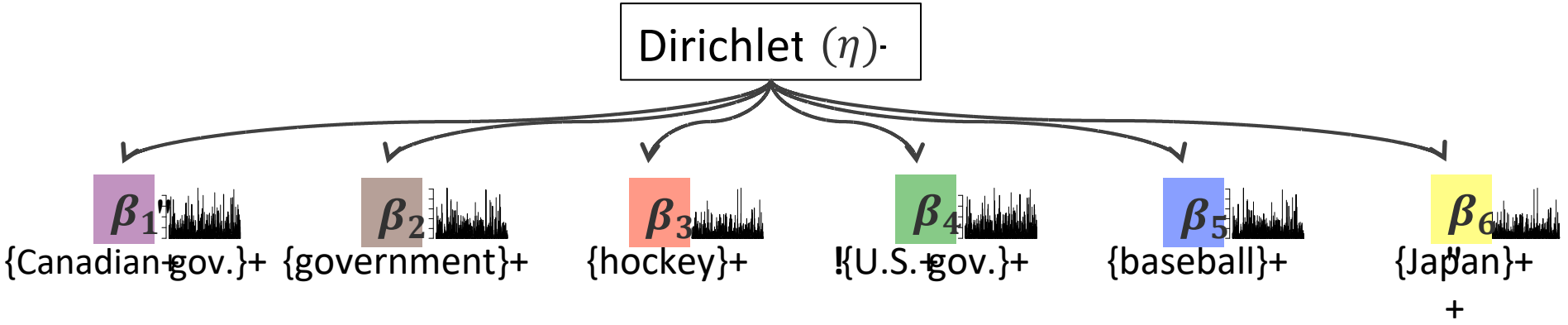

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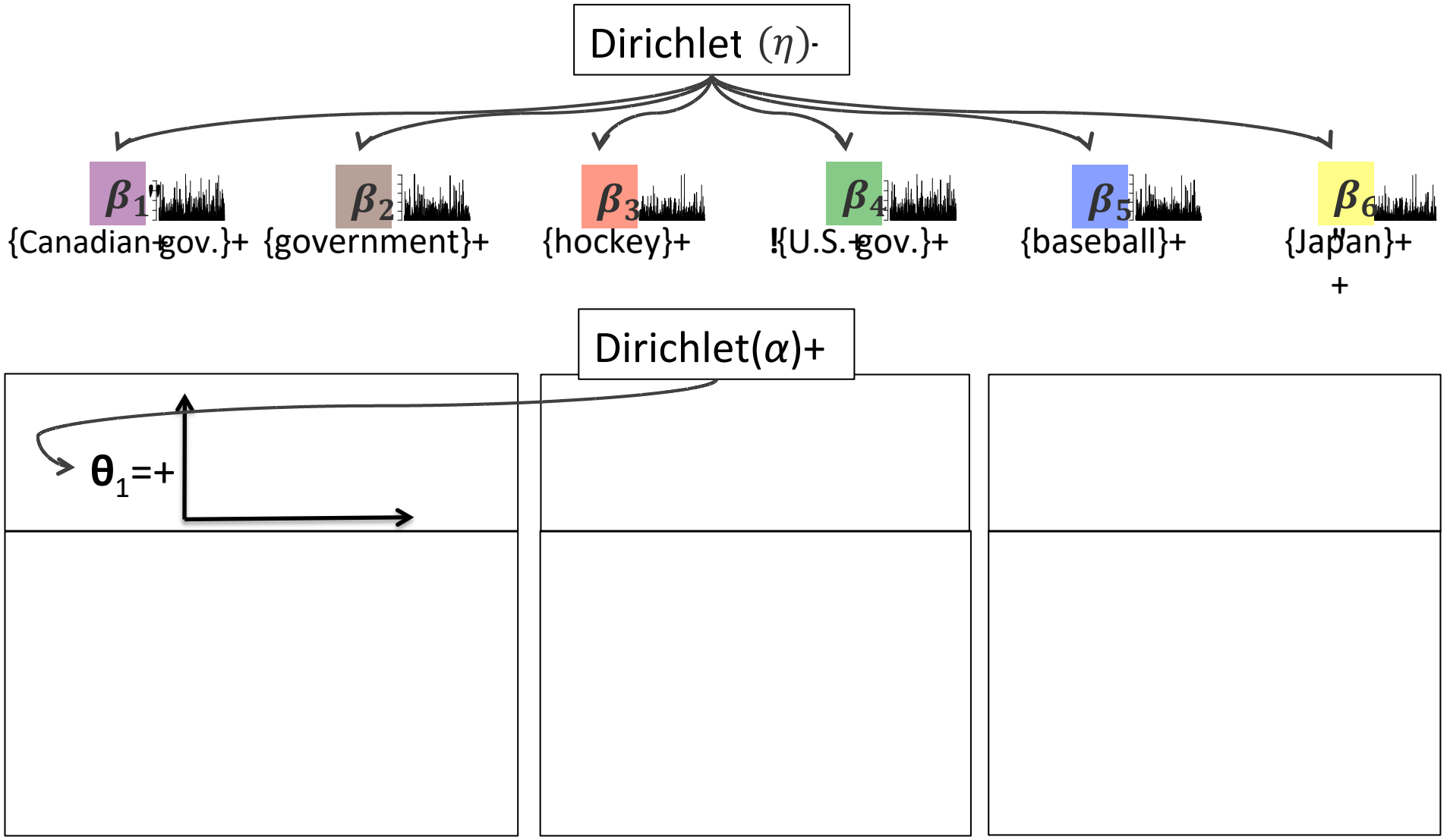

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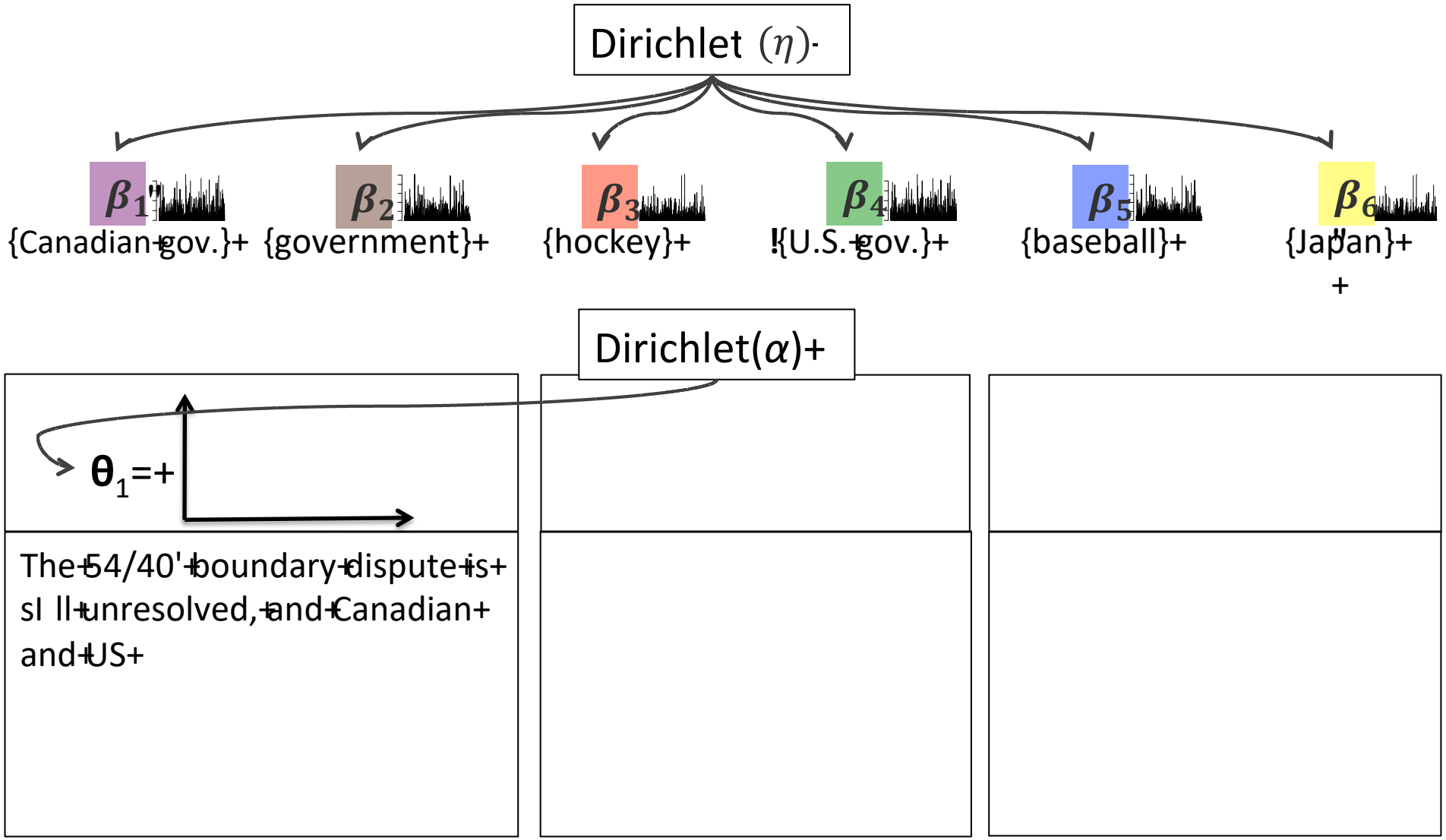

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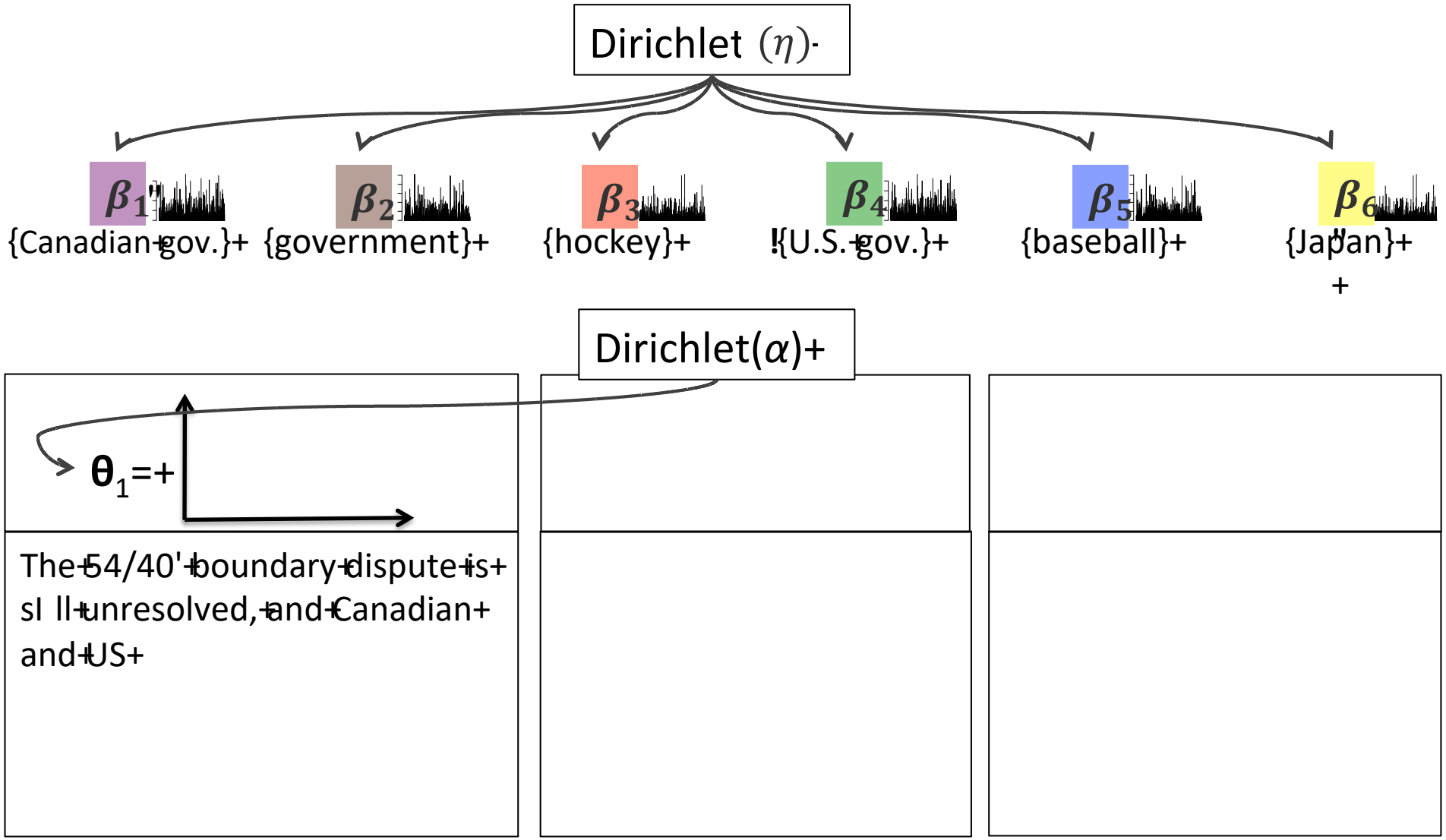

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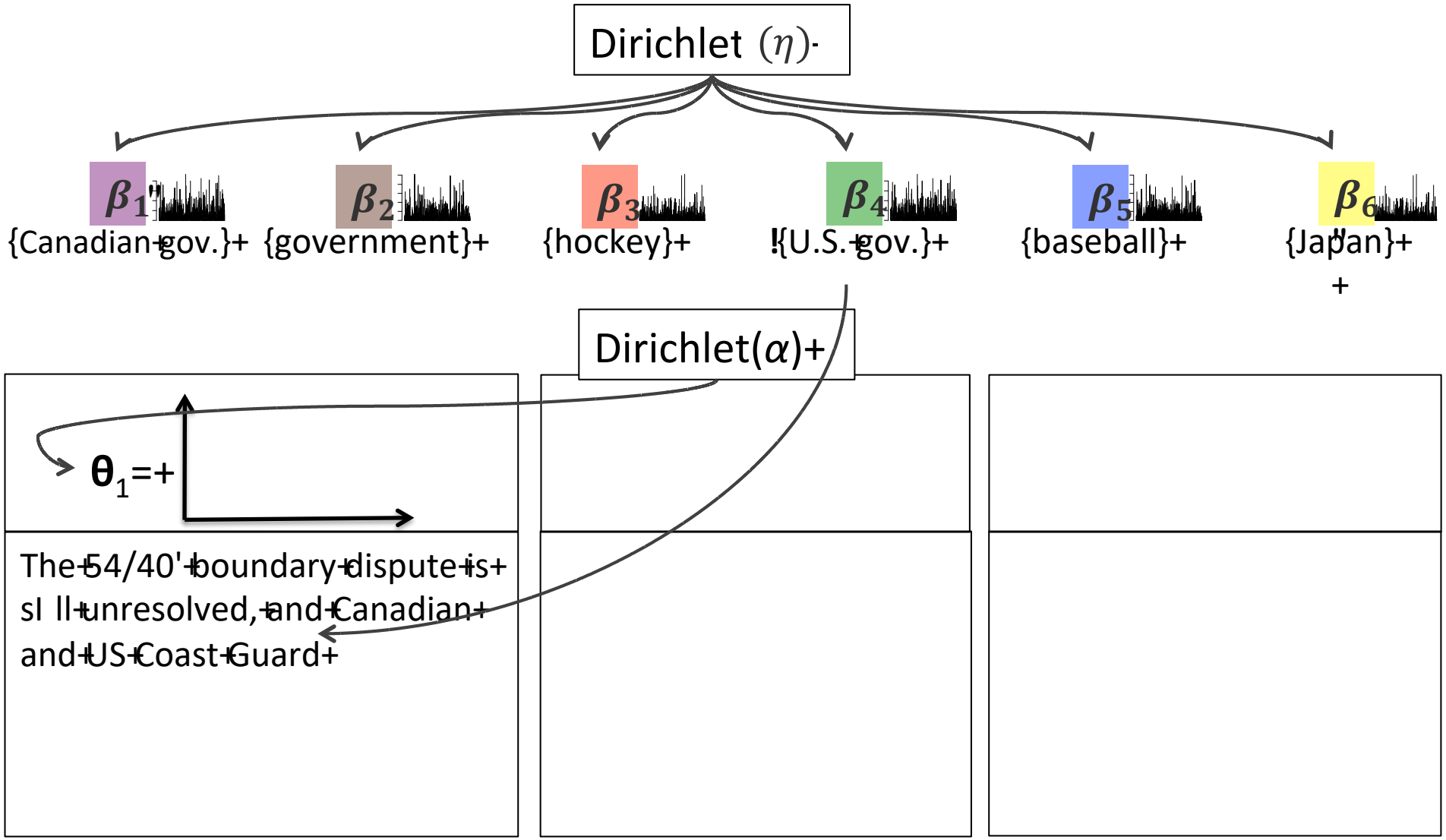

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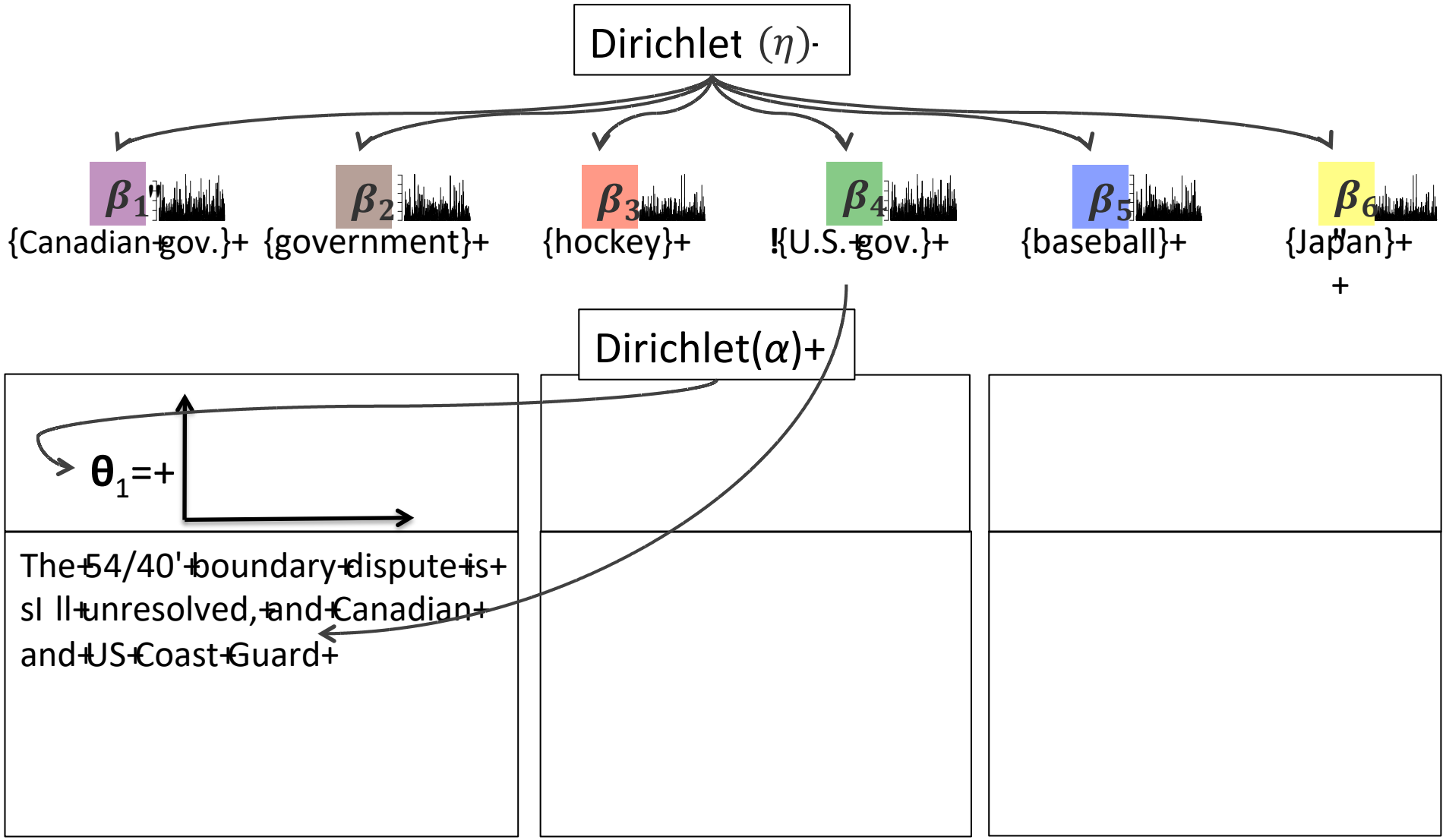

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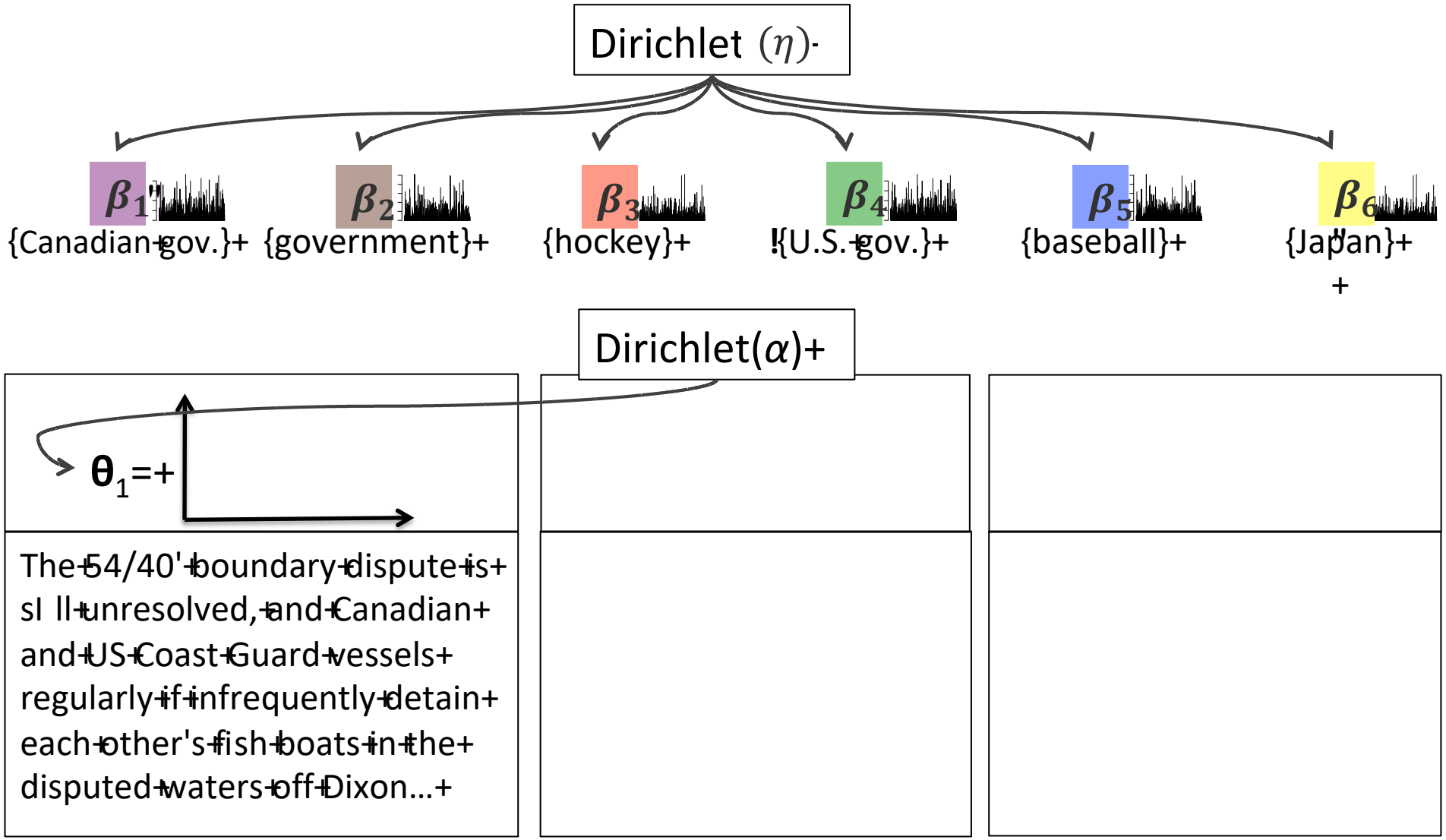

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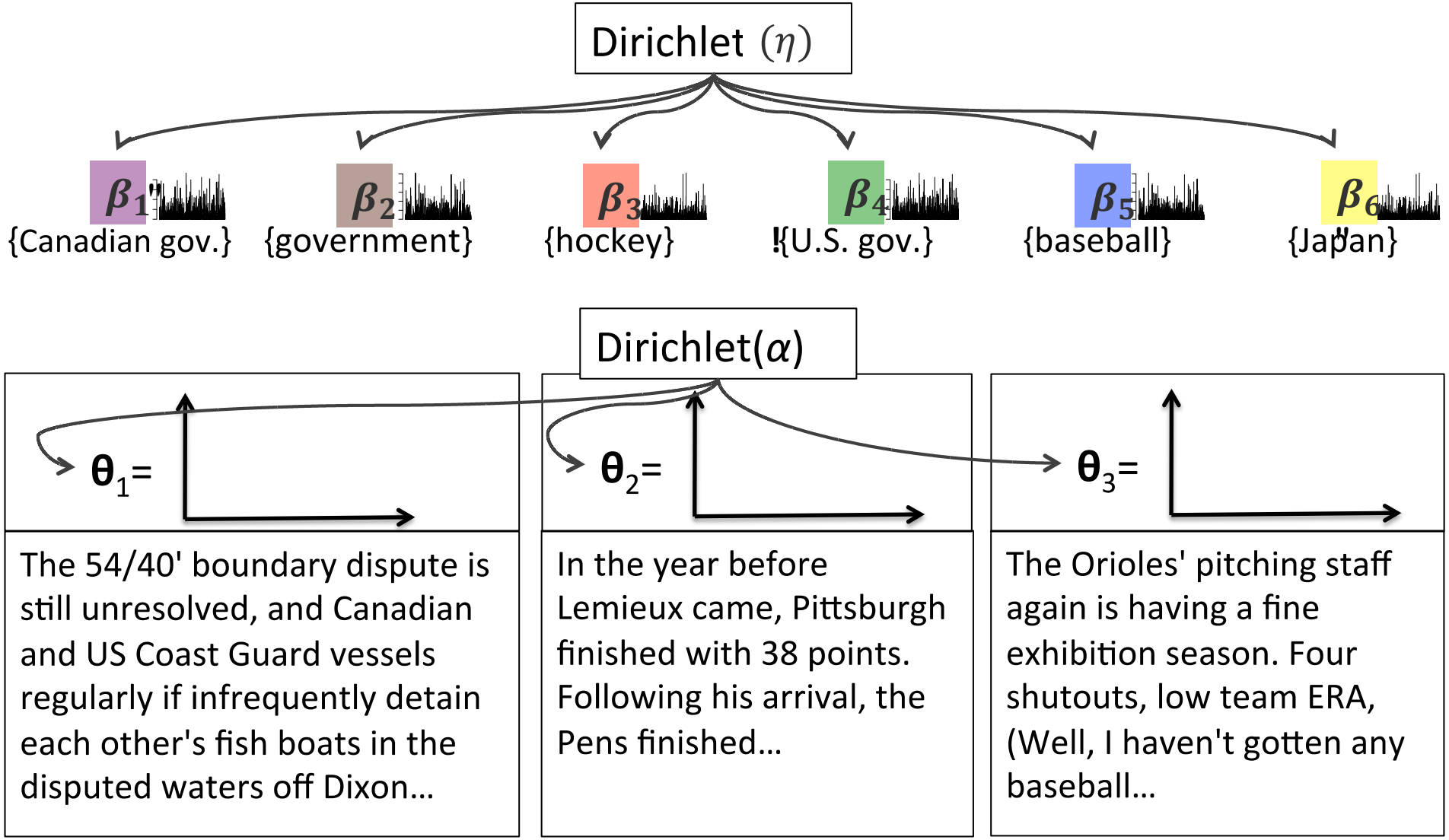

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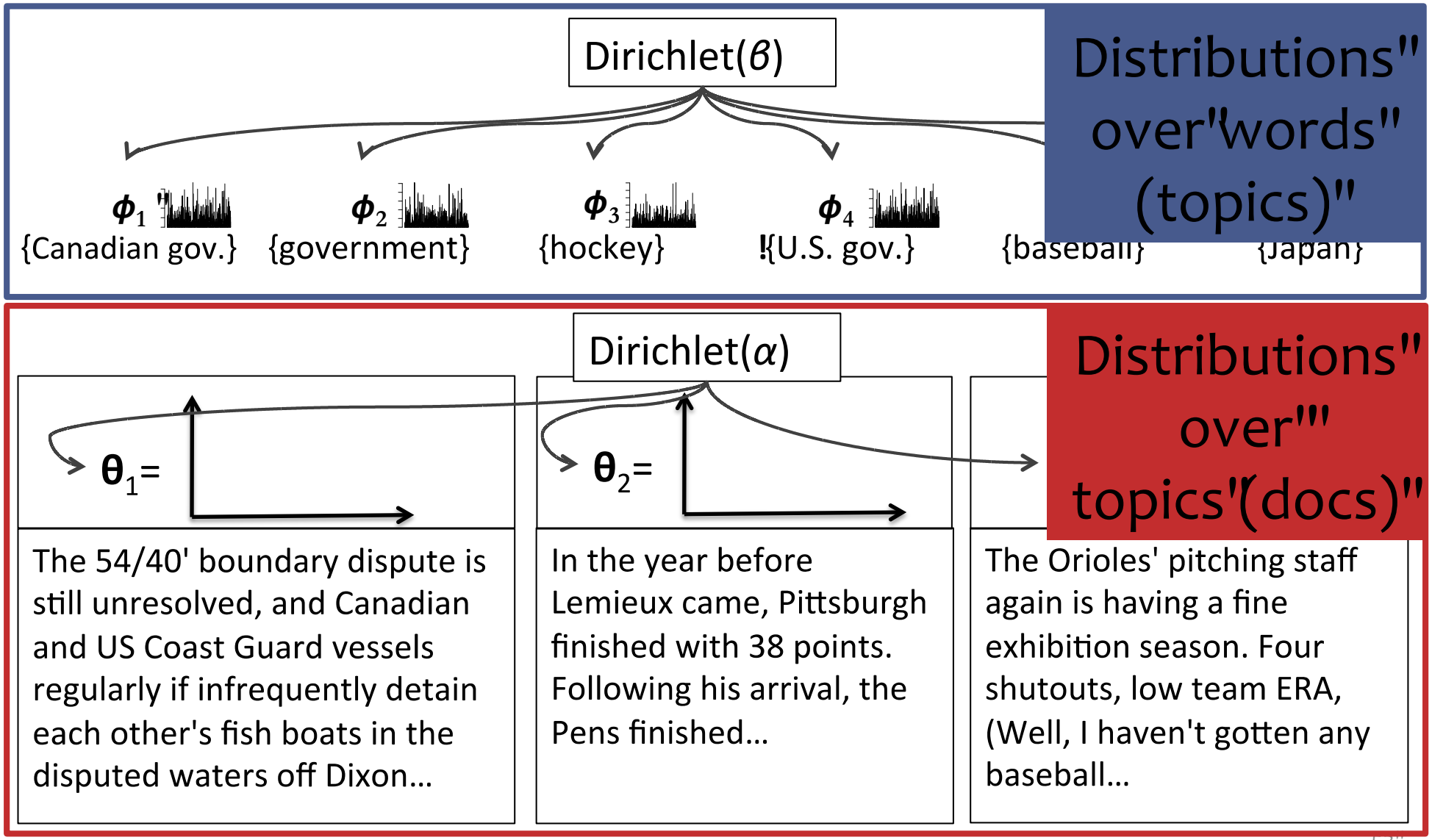
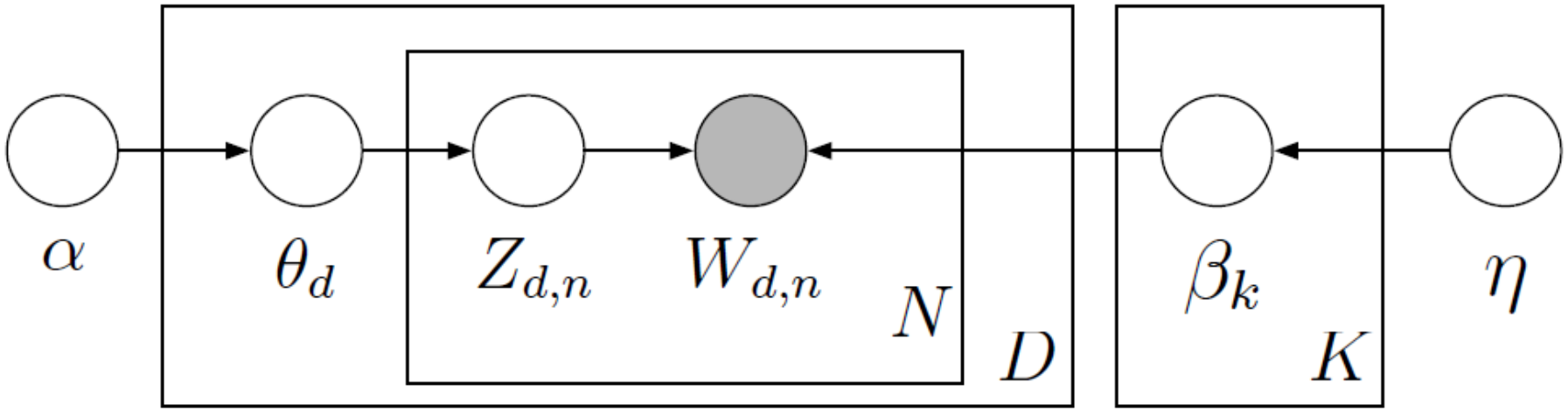


Joint Distribution for LDA



- Joint distribution of latent variables and documents is:

$$p(\boldsymbol{\beta}_{1:K}, \boldsymbol{z}_{1:D}, \boldsymbol{\theta}_{1:D}, \boldsymbol{w}_{1:D} | \alpha, \eta) =$$

$$\prod_{i=1}^{K} p(\beta_i | \eta) \prod_{d=1}^{D} p(\theta_d | \alpha) \left(\prod_{n=1}^{N} p(z_{d,n} | \theta_d) p(w_{d,n} | \beta_{1:K}, z_{d,n}) \right)$$

Questions?