DSC190: Machine Learning with Few Labels

Reinforcement Learning

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Outline

Reinforcement learning

Presentations

- **Kaijie Zhang:** Addition is All You Need for Energy-efficient Language Models
- **Yunshan Li:** Dance Dance Convolution
- **Runyi Yan:** ??
- **Brandon Chiou:** Scaling Rectified Flow Transformers for High-Resolution Image Synthesis

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Putting it together: Deep Q-Learning with Experience Replay

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Initialize action-value function Q with random weights

← Initialize replay memory, Q-network

for episode $= 1, M$ **do**

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for episode $= 1, M$ **do** ← Play M episodes (full games)
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Initialize state (starting game screen pixels) at the beginning of each episode

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For each timestep t of the game

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With small probability, select a random action (explore), otherwise select greedy action from current policy

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Take the action (a_t), and observe the reward r_t and next state s_{t+1}

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Experience Replay: Sample a random minibatch of transitions from replay memory and perform a gradient descent step

Summary so far

- Q-learning:
  - Bellman equation
  - Value-based RL
  - Off-policy RL

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a\sim\rho(\cdot)}\left[(y_i - Q(s,a;\theta_i))^2\right]$

where $y_i = \mathbb{E}_{s'\sim\mathcal{E}}\left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s,a\right]$

- Next: Policy gradient
  - Policy-based RL
  - On-policy RL

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right]$$

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We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

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How can we do this?

Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}[r(\tau)]$$
$$= \int_\tau r(\tau)p(\tau;\theta)\mathrm{d}\tau$$

Where $r(\tau)$ is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \ldots)$

REINFORCE algorithm

Expected reward: $$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$$
$$= \int_{\tau} r(\tau)p(\tau;\theta)\mathrm{d}\tau$$

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Expected reward: $$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$$
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Now let's differentiate this: $$\nabla_\theta J(\theta) = \int_{\tau} r(\tau)\nabla_\theta p(\tau;\theta)\mathrm{d}\tau$$

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Intractable! Gradient of an expectation is problematic when p depends on θ

Question: How to estimate the gradient?

REINFORCE algorithm

Expected reward: $$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$$
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We can use a nice trick: $$\nabla_\theta p(\tau;\theta) = p(\tau;\theta)\frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)$$

REINFORCE algorithm

Expected reward:
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$$\nabla_\theta p(\tau;\theta) = p(\tau;\theta)\frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)$$

If we inject this back:

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right)p(\tau;\theta)\mathrm{d}\tau$$
$$= \mathbb{E}_{\tau \sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

Can estimate with Monte Carlo sampling

REINFORCE algorithm

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau$$
$$= \mathbb{E}_{\tau\sim p(\tau;\theta)}\left[r(\tau)\nabla_\theta \log p(\tau;\theta)\right]$$

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

REINFORCE algorithm

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We have: $p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

Thus: $\log p(\tau;\theta) = \sum_{t\geq 0} \log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)$

REINFORCE algorithm

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau$$
$$= \mathbb{E}_{\tau\sim p(\tau;\theta)} \left[r(\tau)\nabla_\theta \log p(\tau;\theta)\right]$$

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Thus: $\log p(\tau;\theta) = \sum_{t\geq 0} \log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)$

And when differentiating: $\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$

Doesn't depend on transition probabilities!

Intuition

Gradient: $$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} \left[r(\tau) \nabla_\theta \log p(\tau;\theta) \right]$$

Interpretation:

- If r(τ) is high, push up the probabilities of the actions seen
- If r(τ) is low, push down the probabilities of the actions seen

Intuition

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Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

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- If r(τ) is high, push up the probabilities of the actions seen
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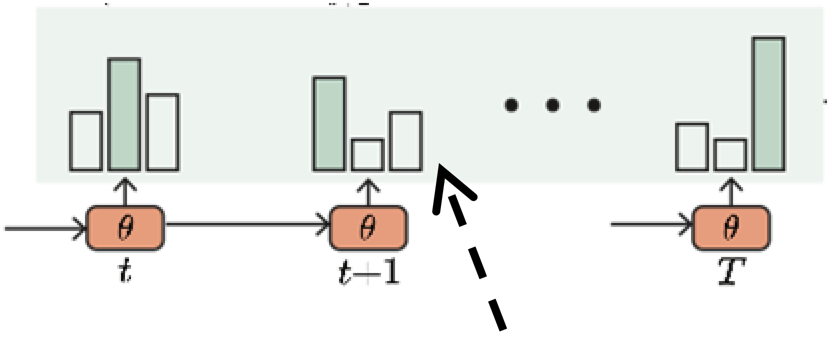
However, this also suffers from high variance because **credit assignment** is really hard.

RL for LLMs

RL for Text Generation: Formulation



- (Autoregressive) text generation model:

Sentence $\boldsymbol{y} = (y_0, \ldots, y_T)$

$\pi_\theta(y_t \mid \boldsymbol{y}_{<t}) = \text{softmax}(\ f_\theta(y_t|\boldsymbol{y}_{<t})\ )$

logits

In RL terms:

trajectory, τ

action, a_t

state, $\boldsymbol{s}_t$

policy $\pi_\theta(a_t \mid \boldsymbol{s}_t\ )$

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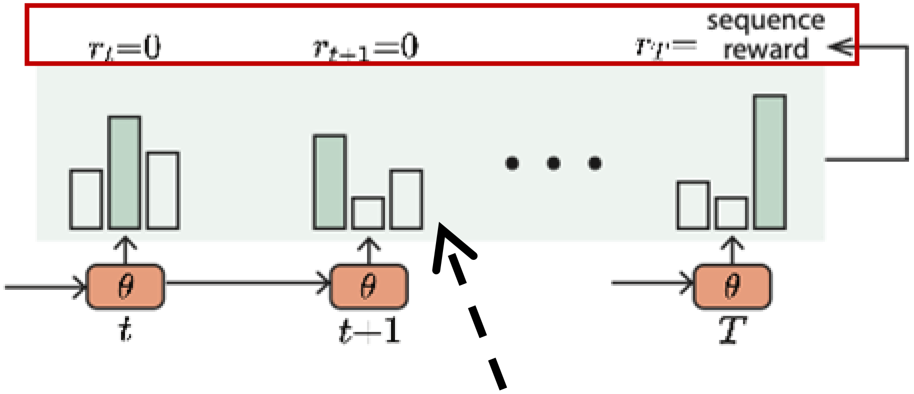

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In RL terms: trajectory, τ action, a_t state, $\boldsymbol{s}_t$ policy $\pi_\theta(a_t \mid \boldsymbol{s}_t\ )$

- Reward $r_t = r(\boldsymbol{s}_t, a_t)$
  - Often **sparse**: $r_t = 0$ for $t < T$
- The general RL objective: maximize cumulative reward $J(\pi) = \mathbb{E}_{\tau \sim \pi}\left[\sum_{t=0}^{T} \gamma^t r_t\right]$
- Q-function: expected *future* reward of taking action a_t in state $\boldsymbol{s}_t$

$$Q^\pi(\boldsymbol{s}_t, a_t) = \mathbb{E}_\pi\left[\ \sum_{t'=t}^{T} \gamma^{t'}\ r_{t'} \mid\ \boldsymbol{s}_t, a_t\ \right]$$

Presentations

Questions?