

DSC190: Machine Learning with Few Labels

Reinforcement Learning

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Lecture 21, November 18, 2024

UC San Diego

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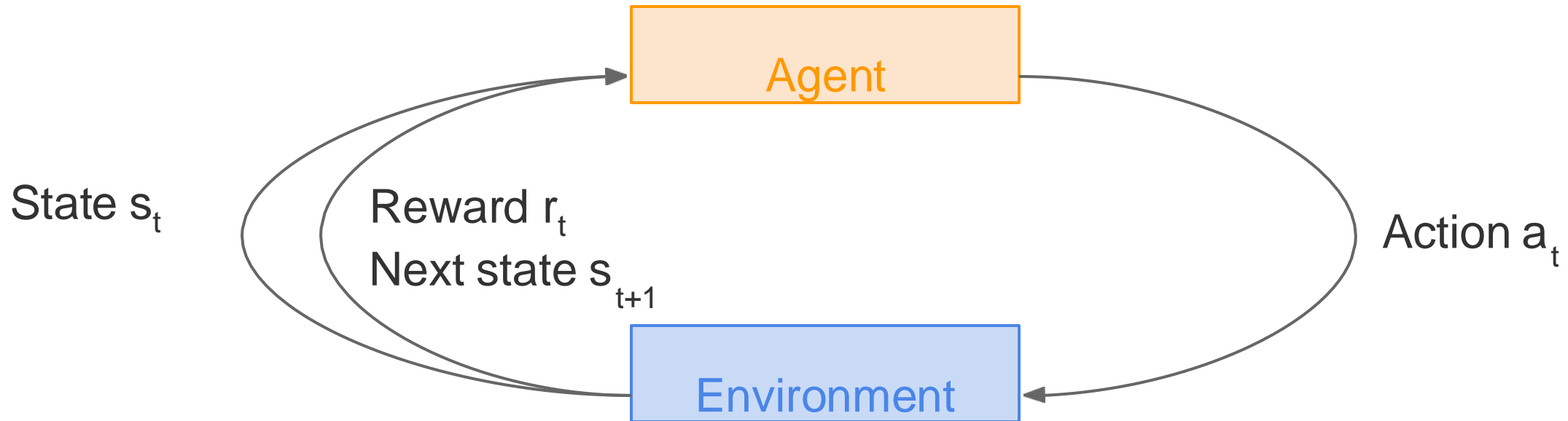
Outline

Reinforcement learning

Presentations

- **Jiangqi Wu:** Post-training Quantization for Neural Networks with Provable Guarantees
- **Jessica Song:** Diffusion Model
- **Gautham Kishore:** Code Translation with Compiler Representations
- **Aleck Wu:** UMAP: Uniform Manifold Approximation for Dimension Reduction

How can we mathematically formalize the RL problem?



Recap: Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

How good is a state-action pair?

The **Q-value function** at state s and action a , is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s, a) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Recursive expansion of Q :

Recap: Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Q^* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') \mid s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s', a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') | s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \infty$

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What's the problem with this?

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What's the problem with this?

Not scalable. Must compute $Q(s, a)$ for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solving for the optimal policy

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Question: how would you solve the issue?

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Question: how would you solve the issue?

Solution: use a function approximator to estimate $Q(s,a)$. E.g. a neural network!

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

Solving for the optimal policy: Q-learning

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If the function approximator is a deep neural network => **deep q-learning!**

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning!**

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

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Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} \left[(y_i - Q(s, a; \theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

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Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$

Case Study: Playing Atari Games



Objective: Complete the game with the highest score

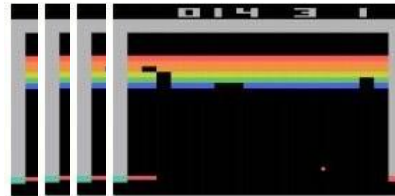
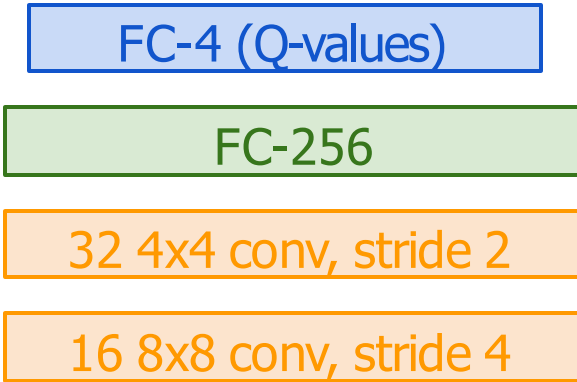
State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

Q-network Architecture

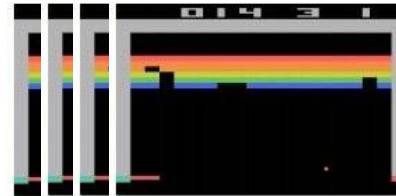
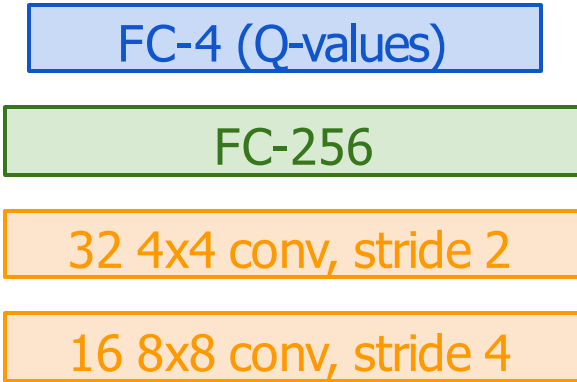
$Q(s, a; \theta)$:
neural network
with weights θ



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

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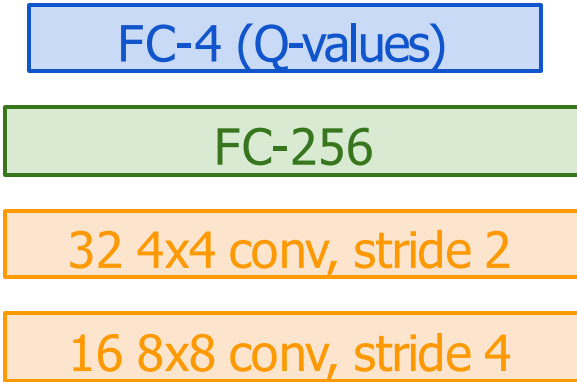


← Input: state s_t

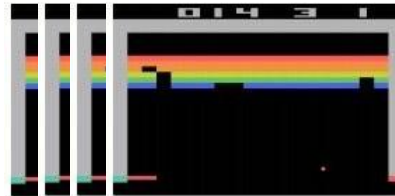
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← Familiar conv layers,
FC layer



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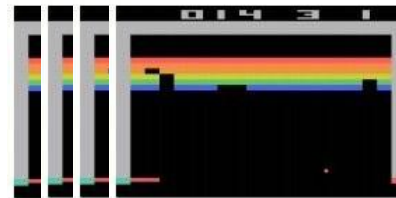
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FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4



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← Last FC layer has 4-d
output (if 4 actions),
corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$,
 $Q(s_t, a_4)$

Q-network Architecture

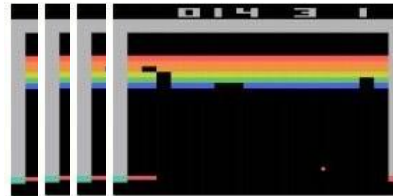
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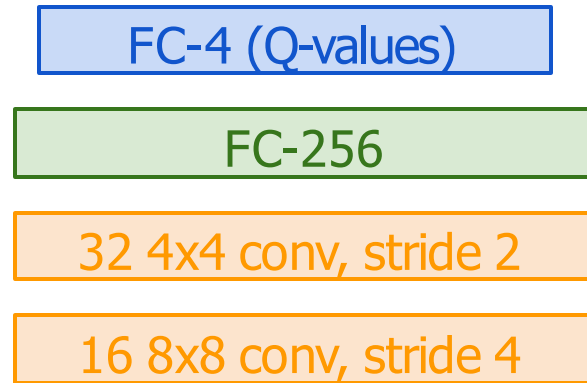
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Number of actions between 4-18
depending on Atari game

Q-network Architecture

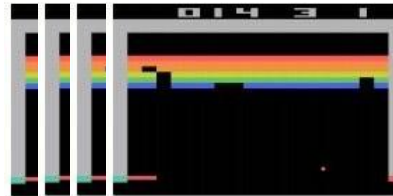
$Q(s, a; \theta)$:
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A single feedforward pass
to compute Q-values for all
actions from the current
state => efficient!



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Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

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Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

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Each transition can also contribute to multiple weight updates
=> greater data efficiency

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

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← Initialize replay memory, Q-network

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← Play M episodes (full games)

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Initialize state
(starting game
screen pixels) at the
beginning of each
episode

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For each timestep t
of the game

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end for

← With small probability, select a random action (explore), otherwise select greedy action from current policy

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← Take the action (a_t), and observe the reward r_t and next state s_{t+1}

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← Store transition in replay memory

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end for

← Experience Replay:
Sample a random minibatch of transitions from replay memory and perform a gradient descent step

Summary so far

- Q-learning:
 - Bellman equation
 - Value-based RL
 - Off-policy RL

- Next: Policy gradient
 - Policy-based RL
 - On-policy RL

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} [(y_i - Q(s, a; \theta_i))^2]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} [r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a]$

Policy Gradients

What is a problem with Q-learning?

The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

Policy Gradients

What is a problem with Q-learning?

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Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand
Can we learn a policy directly?

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

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Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$\begin{aligned} J(\theta) &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)] \\ &= \int_{\tau} r(\tau) p(\tau; \theta) d\tau \end{aligned}$$

Where $r(\tau)$ is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \dots)$

REINFORCE algorithm

Expected reward:

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Now let's differentiate this: $\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$

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Intractable! Gradient of an expectation is problematic when p depends on θ

Question: How to estimate the gradient?

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If we inject this back:

$$\begin{aligned} \nabla_{\theta} J(\theta) &= \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau \\ &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)] \end{aligned}$$

Can estimate with Monte Carlo sampling

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$

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Thus: $\log p(\tau; \theta) = \sum_{t \geq 0} \log p(s_{t+1} | s_t, a_t) + \log \pi_{\theta}(a_t | s_t)$

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Therefore when sampling a trajectory τ , we can estimate $J(\theta)$ with

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Intuition

Gradient estimator: $\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

Interpretation:

- If $r(\tau)$ is high, push up the probabilities of the actions seen
- If $r(\tau)$ is low, push down the probabilities of the actions seen

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However, this also suffers from high variance because **credit assignment** is really hard.

Presentations

Questions?