## **DSC250: Advanced Data Mining**

## Topic Models

Zhiting Hu Lecture 5, October 12, 2023



HALICIOĞLU DATA SCIENCE INSTITUTE

### Outline

- Representations of Text and Topics
- Topic Model v1: Multinomial Mixture Model
- Topic Model v2: Probabilistic Latent Semantic Analysis (pLSA)
- Topic Model v3: Latent Dirichlet Allocation (LDA)

Slides adapted from:

- Y. Sun, CS 247: Advanced Data Mining
- M. Gormley, 10-701 Introduction to Machine Learning

### Motivation

Suppose you're given a massive corpora and asked to carry out the following tasks

- **Organize** the documents into **thematic categories**
- **Describe** the evolution of those categories over time
- Enable a domain expert to **analyze and understand** the content
- Find **relationships** between the categories
- Understand how **authorship** influences the content



### Motivation

Suppose you're given a massive corpora and asked to carry out the following tasks

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#### **Topic Modeling:**

A method of (usually unsupervised) discovery of latent or hidden structure in a corpus

- Applied primarily to text corpora, but **techniques are more general**
- Provides a **modeling toolbox**
- Has prompted the exploration of a variety of new inference methods to accommodate large-scale datasets

#### Topic Modeling:



1 8 16 26 Figuré 1<sup>t</sup>. At right are the top 15 most frequent words from the most frequent topics found in this appicle.

is drawn from one of the topics (step #2b), where the selected topic is chosen from the per-document distribution over topics (step #2a).<sup>2</sup> Figure from (Blei, 2011), Shows topics and top words learned In the example article, the distribution over topics would place probability on genetics, automatics and represent topics and compared over topics one of those three topics. Notice that the next article in the collection might be about *data analysis* and *neuroscience*; its distribution over topics would place probability on those two topics. This is the distinguishing characteristic of latent Dirichlet allocation—all the documents in the

[15-780, Kolter]

### **Topic Modeling: Examples**

**Dirichlet-multinomial regression (DMR) topic model on ICML** (Mimno & McCallum, 2008)

Topic 0 [0.152]



#### Topic 54 [0.051]



#### Topic 99 [0.066]



problem, optimization, problems, convex, convex optimization, linear, semidefinite programming, formulation, sets, constraints, proposed, margin, maximum margin, optimization problem, linear programming, programming, procedure, method, cutting plane, solutions

decision trees, trees, tree, decision tree, decision, tree ensemble, junction tree, decision tree learners, leaf nodes, arithmetic circuits, ensembles modts, skewing, ensembles, anytime induction decision trees, trees trees, random forests, objective decision trees, tree learners, trees grove, candidate split

inference, approximate inference, exact inference, markov chain, models, approximate, gibbs sampling, variational, bayesian, variational inference, variational bayesian, approximation, sampling, methods, exact, bayesian inference, dynamic bayesian, process, mcmc, efficient

http://www.cs.umass.edu/~mimno/icml100.html



#### **Other Applications of Topic Models**

Spacial LDA

(Wang & Grimson, 2007)



### **Other Applications of Topic Models**

• Word Sense Induction

(Brody & Lapata, 2009)

Senses of <i>drug</i> (WSJ)				
1. U.S., administration, federal, against, war, dealer				
2. patient, people, problem, doctor, company, abuse				
3. company, million, sale, maker, stock, inc.				
4. administration, food, company, approval, FDA				

Senses of *drug* (BNC) 1. patient, treatment, effect, anti-inflammatory 2. alcohol, treatment, patient, therapy, addiction 3. patient, new, find, effect, choice, study 4. test, alcohol, patient, abuse, people, crime 5. trafficking, trafficker, charge, use, problem 6. abuse, against, problem, treatment, alcohol 7. people, wonder, find, prescription, drink, addict 8. company, dealer, police, enforcement, patient

• Selectional Preference<sup>7. people, wonder, find, prescription, drink, addi 8. company, dealer, police, enforcement, patient</sup>

#### (Ritter et al., 2010)

Topic t	Arg1	Relations which assign	Arg2
		highest probability to $t$	
18	The residue - The mixture - The reaction	was treated with, is	EtOAc - CH2Cl2 - H2O - CH.sub.2Cl.sub.2
	mixture - The solution - the mixture - the re-	treated with, was	- H.sub.2O - water - MeOH - NaHCO3 -
	action mixture - the residue - The reaction -	poured into, was	Et2O - NHCl - CHCl.sub.3 - NHCl - drop-
	the solution - The filtrate - the reaction - The	extracted with, was	wise - CH2Cl.sub.2 - Celite - Et.sub.2O -
	product - The crude product - The pellet -	purified by, was di-	Cl.sub.2 - NaOH - AcOEt - CH2C12 - the
	The organic layer - Thereto - This solution	luted with, was filtered	mixture - saturated NaHCO3 - SiO2 - H2O
	- The resulting solution - Next - The organic	through, is disolved in,	- N hydrochloric acid - NHCl - preparative
	phase - The resulting mixture - C. )	is washed with	HPLC - to0 C

#### Text Data

- Word/term
- Document
  - A sequence of words
- Corpus
  - A collection of

documents



#### **Represent a Document**

#### Most common way: Bag-of-Words

- Ignore the order of words
- keep the count
- c1: Human machine interface for Lab ABC computer applications
- c2: A survey of user opinion of computer system response time
- c3: The EPS user interface management system
- c4: System and human system engineering testing of EPS
- c5: Relation of user-perceived response time to error measurement
- m1: The generation of random, binary, unordered trees
- m2: The intersection graph of paths in trees
- m3: Graph minors IV: Widths of trees and well-quasi-ordering
- m4: Graph minors: A survey

	CT	C2	СЗ	C4	C5	mı	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

#### Vector space model

#### Represent a Document

- Represent the doc as a vector where each entry corresponds to a different word and the number at that entry corresponds to how many times that word was present in the document (or some function of it)
  - Number of words is huge
  - Select and use a smaller set of words that are of interest
  - E.g. uninteresting words: 'and', 'the' 'at', 'is', etc. These are called <u>stop-words</u>
  - <u>Stemming:</u> remove endings. E.g. 'learn', 'learning', 'learnable', 'learned' could be substituted by the single stem 'learn'
  - Other simplifications can also be invented and used
  - The set of different remaining words is called <u>dictionary</u> or <u>vocabulary</u>. Fix an ordering of the terms in the dictionary so that you can operate them by their index.
  - Can be extended to bi-gram, tri-gram, or so

### Limitations of Bag-of-Words

- Dimensionality
  - High dimensionality
- Sparseness
  - Most of the entries are zero
- Shallow representation
  - The vector representation does not capture semantic relations between words

Ex: "Tom loves Kate."

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#### lopics

#### **Represent a Topic**

#### • A topic is represented by a word distribution

#### • Relate to an issue

0.0439	drug	0.0672	cells	0.0675	sequence	0.0818	years	0.156
0.0375	patients	0.0493	stem	0.0478	sequences	0.0493	million	0.0556
0.0279	drugs	0.0444	human	0.0421	genome	0.033	ago	0.045
0.0233	clinical	0.0346	cell	0.0309	dna	0.0257	time	0.0317
0.0232	treatment	0.028	gene	0.025	sequencing	0.0172	age	0.0243
0.0214	trials	0.0277	tissue	0.0185	map	0.0123	year	0.024
0.0137	therapy	0.0213	cloning	0.0169	genes	0.0122	record	0.0238
0.0131	trial	0.0164	transfer	0.0155	chromosome	0.0119	early	0.0233
0.0109	disease	0.0157	blood	0.0113	regions	0.0119	billion	0.0177
0.01	medical	0.00997	embryos	0.0111	human	0.0111	history	0.0148
0.0983	male	0.0558	theory	0.0811	immune	0.0909	stars	0.0524
0.0561	females	0.0541	physics	0.0782	response	0.0375	star	0.0458
0.0431	female	0.0529	physicists	0.0146	system	0.0358	astrophys	0.0237
0.0381	males	0.0477	einstein	0.0142	responses	0.0322	mass	0.021
0.025	sex	0.0339	university	0.013	antigen	0.0263	disk.	0.0173
0.0214	reproductive	0.0172	gravity	0.013	antigens	0.0184	black	0.0161
0.0196	offspring	0.0168	black	0.0127	immunity	0.0176	gas	0.0149
0.0165	sexual	0.0166	theories	0.01	immunology	0.0145	stellar	0.0127
0.0163	reproduction	0.0143	aps	0.00987	antibody	0.014	astron	0.0125
0.0145	eggs	0.0138	matter	0.00954	autoimmune	0.0128	hole	0.00824
	0.0439 0.0375 0.0279 0.0233 0.0232 0.0214 0.0137 0.0131 0.0109 0.01 0.0983 0.0561 0.0431 0.025 0.0214 0.0196 0.0165 0.0163 0.0145	0.0439         drug           0.0375         patients           0.0279         drugs           0.0233         clinical           0.0232         treatment           0.0214         trials           0.0137         therapy           0.0131         trial           0.0109         disease           0.01         medical           0.0561         females           0.0431         female           0.025         sex           0.0214         reproductive           0.0155         sexual           0.0165         sexual           0.0165         sexual           0.0165         sexual	0.0439         drug         0.0672           0.0375         patients         0.0493           0.0279         drugs         0.0444           0.0233         clinical         0.0346           0.0232         treatment         0.028           0.0214         trials         0.0277           0.0137         therapy         0.0213           0.0137         therapy         0.0213           0.0137         therapy         0.0213           0.0137         therapy         0.0213           0.0131         trial         0.0164           0.0109         disease         0.0157           0.01         medical         0.00997           0.0983         male         0.0558           0.0561         females         0.0541           0.0431         females         0.0477           0.025         sex         0.0339           0.0214         reproductive         0.0172           0.0196         offspring         0.0168           0.0165         sexual         0.0166           0.0165         sexual         0.0143           0.0145         eggs         0.0138	0.0439         drug         0.0672         cells           0.0375         patients         0.0493         stem           0.0279         drugs         0.0444         human           0.0233         clinical         0.0346         cell           0.0233         treatment         0.028         gene           0.0214         trials         0.0277         tissue           0.0137         therapy         0.0213         cloning           0.0137         thial         0.0164         transfer           0.0131         trial         0.0164         transfer           0.0109         disease         0.0157         blood           0.01         medical         0.00997         embryos           0.0561         females         0.0558         theory           0.0431         female         0.0529         physicists           0.0381         males         0.0477         einstein           0.025         sex         0.0339         university           0.0214         reproductive         0.0172         gravity           0.0165         sexual         0.0166         theories           0.0165         sexual         0.016	0.0439         drug         0.0672         cells         0.0675           0.0375         patients         0.0493         stem         0.0478           0.0279         drugs         0.0444         human         0.0421           0.0233         clinical         0.0346         cell         0.0309           0.0232         treatment         0.028         gene         0.025           0.0214         trials         0.0277         tissue         0.0185           0.0137         therapy         0.0213         cloning         0.0169           0.0131         trial         0.0164         transfer         0.0155           0.0109         disease         0.0157         blood         0.0113           0.01         medical         0.00997         embryos         0.0111           0.0983         male         0.0558         theory         0.0811           0.0561         females         0.0477         einstein         0.0142           0.025         sex         0.0339         university         0.013           0.025         sex         0.0339         university         0.013           0.025         sex         0.0339         university </td <td><math display="block"> \begin{array}{c ccccccccccccccccccccccccccccccccccc</math></td> <td></td> <td></td>	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		

TOPIC 42

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#### TOPIC 45 consist identified implementing annu ally . implement in created under september in formation ECOn Orn i nitiated pproved including recen shated building 1 rogram ine basic offers our ce miles fall as billionannual Source miles fall as billionannual Source miles fall as billionannual Source miles as a state of the source miles and the source miles and state as a source miles and the source miles and state as a source miles and the source miles and state as a source miles and the source miles and state as a source miles and the source miles and state as a source miles and the source miles and state as a source miles and the source miles and the source miles and state as a source miles and the source addition approximately stimated service in provement ilitate Years total division and prated found operates work in Sompletion

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#### TOPIC 43

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#### TOPIC 49

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### **Topic Models**

#### Topic modeling

- Get topics automatically from a corpus
- Assign documents to topics automatically
- Most frequently used topic models
  - pLSA
  - LDA

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHEB
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

The William Randolph Hearst Foundation will give \$1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be \$200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive \$400,000 each. The Juilliard School, where music and the performing arts are taught, will get \$250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual \$100,000 donation, too.

#### Notations

- Word, document, topic
  - $\circ w, d, z$
- Word count in document:
  - c(w,d): number of times word w occurs in document d
  - or  $x_{dn}$ : number of times the *n*th word in the vocabulary occurs in document d
- Word distribution for each topic (  $\beta_z$  )
  - $\beta_{zw}$ : p(w|z)



### **Recap: Multinomial distribution**

- Multinomial distribution
  - Discrete random variable x that takes one of M values  $\{1, ..., M\}$

$$\circ p(\boldsymbol{x}=i) = \pi_{i}, \qquad \sum_{i} \pi_{i} = 1$$

- Out of *n* independent trials, let  $k_i$  be the number of times x = i was observed
- The probability of observing a vector of occurrences  $\mathbf{k} = [k_1, ..., k_M]$  is given by the *multinomial distribution* parametrized by  $\boldsymbol{\pi}$

$$p(\mathbf{k}|\boldsymbol{\pi}, \mathbf{n}) = p(k_1, \dots, k_m | \pi_1, \dots, \pi_m, \mathbf{n}) = \frac{\mathbf{n}!}{k_1! k_2! \dots k_m!} \prod_{i=1}^{n} \pi_i^{k_i}$$

- E.g., describing a text document by the frequency of occurrence of every distinct word
- For n = 1, a.k.a. categorical distribution
  - $p(\boldsymbol{x} = i \mid \boldsymbol{\pi}) = \pi_i$
- In  $\mathbf{k} = [k_1, ..., k_M]$ :  $k_i = 1$ , and  $k_j = 0$  for all  $j \neq i \rightarrow a.k.a.$ , one-hot representation of i [CSC2515, Wang] 17

- For documents with bag-of-words representation
  - $x_d = (x_{d1}, x_{d2}, ..., x_{dN}), x_{dn}$  is the number of words for nth word in the vocabulary
- Generative model

For documents with bag-of-words representation

•  $x_d = (x_{d1}, x_{d2}, ..., x_{dN}), x_{dn}$  is the number of words for nth word in the vocabulary

Generative model

Formulating the statistical relationship between words, documents and latent topics as a generative process describing how documents are created

- For documents with bag-of-words representation
  - $x_d = (x_{d1}, x_{d2}, ..., x_{dN}), x_{dn}$  is the number of words for nth word in the vocabulary
- Generative model
  - For each document
    - Sample its cluster label  $z \sim Categorical(\pi)$

•  $\boldsymbol{\pi} = (\pi_1, \pi_2, ..., \pi_K), \pi_k$  is the proportion of jth cluster

•  $p(z=k) = \pi_k$ 

- Sample its word vector  $\mathbf{x}_d \sim multinomial(\boldsymbol{\beta}_z)$ 
  - $\beta_z = (\beta_{z1}, \beta_{z2}, ..., \beta_{zN}), \beta_{zn}$  is the parameter associate with nth word in the vocabulary

• 
$$p(\mathbf{x}_d|z=k) = \frac{(\sum_n x_{dn})!}{\prod_n x_{dn}!} \prod_n \beta_{kn}^{x_{dn}} \propto \prod_n \beta_{kn}^{x_{dn}}$$

Graphical Model

- Plates indicate replicated variables.
- Shaded nodes are observed; unshaded nodes are hidden.

### Generative model

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#### Likelihood Function

# Likelihood Function

$$L = \prod_{d} p(\mathbf{x}_{d}) = \prod_{d} \sum_{k} p(\mathbf{x}_{d}, z = k)$$
$$= \prod_{d} \sum_{k} p(\mathbf{x}_{d} | z = k) p(z = k)$$
$$= \prod_{d} \frac{(\sum_{n} x_{dn})!}{\prod_{n} x_{dn}!} \sum_{k} p(z = k) \prod_{n} \beta_{kn}^{x_{dn}}$$

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Limitations of Multinomial Mixture Model

• All the words in the same documents are sampled from the same topic



• In practice, people switch topics during their writing

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## Limitations of Multinomial Mixture Model Mixture vs. Admixture



Diagrams from Wallach, JHU 2011, slides

documents

#### Topic Model v2: Probabilistic Latent Semantic Analysis (pLSA)

ŝ	ALUS	Dudgets	omuten	Education
	NEW	MILLION	CHILDREN	SCHOOL
	FILM	TAX	WOMEN	STUDENTS
	SHOW	PROGRAM	PEOPLE	SCHOOLS
	MUSIC	BUDGET	CHILD	EDUCATION
	MOVIE	BILLION	YEARS	TEACHERS
	PLAY	FEDERAL	FAMILIES	HIGH
	MUSICAL	YEAR	WORK	PUBLIC
	BEST	SPENDING	PARENTS	TEACHER
	ACTOR	NEW	SAYS	BENNETT
	FIRST	STATE	FAMILY	MANIGAT
	YORK	PLAN	WELFARE	NAMPHY
	OPERA	MONEY	MEN	STATE
	THEATER	PROGRAMS	PERCENT	PRESIDENT
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#### Generative Model for pLSA

- For each position in d,  $n = 1, ..., N_d$ 
  - Generate the topic for the position as  $z_n | d \sim Categorical(\theta_d), i.e., p(z_n = k | d) = \theta_{dk}$ (Note, 1 trial multinomial)
    - Generate the word for the position as

 $w_n | z_n \sim Categorical(\boldsymbol{\beta}_{z_n}), i.e., p(w_n = w | z_n) = \beta_{z_n w}$ 

#### Graphical Model for pLSA



Note: Sometimes, people add parameters such as  $\theta$  and  $\beta$  into the graphical model

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# **Likelihood Function** Probability of a word w $p(w|d,\theta,\beta) = \sum p(w,z=k|d,\theta,\beta)$ $=\sum p(w|z=k,d,\theta,\beta)p(z=k|d,\theta,\beta)=\sum \beta_{kw}\theta_{dk}$ $\prod^{N_d} P(w_1, \cdots, w_{N_d}, d | \theta, \beta, \pi)$ d=1 $= \prod_{i=1}^{N_d} P(d) \left\{ \prod_{i=1}^{N_d} \left( \sum_{i=1}^{N_d} P(z_n = k | d, \theta_d) P(w_n | \beta_k) \right) \right\}$ $= \prod_{l=1}^{N_d} \pi_d \left\{ \prod_{l=1}^{N_d} \left( \sum_{l=1}^{N_d} \theta_{dk} \beta_{kw_n} \right) \right\}$

Likelihood Function • Probability of a word w  $p(w|d,\theta,\beta) = \sum_{k} p(w,z=k|d,\theta,\beta)$  $= \sum_{k} p(w|z=k,d,\theta,\beta)p(z=k|d,\theta,\beta) = \sum_{k} \beta_{kw}\theta_{dk}$ 

Likelihood of a corpus

$$\begin{split} &\prod_{d=1}^{n} P(w_1, \cdots, w_{N_d}, d | \theta, \beta, \pi) \\ &= \prod_{d=1}^{n} P(d) \left\{ \prod_{n=1}^{N_d} \left( \sum_k P(z_n = k | d, \theta_d) P(w_n | \beta_k) \right) \right\} \\ &= \prod_{d=1}^{n} \pi_d \left\{ \prod_{n=1}^{N_d} \left( \sum_k \theta_{dk} \beta_{kw_n} \right) \right\} \\ & \pi_d \text{ is usually considered as uniform, i.e., 1/M} \end{split}$$

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**Re-arrange the Likelihood Function** 

Group the same word from different positions together

$$\max \log L = \sum_{dw} c(w, d) \log \sum_{z} \theta_{dz} \beta_{zw}$$
  
s.t.  $\sum_{z} \theta_{dz} = 1$  and  $\sum_{w} \beta_{zw} = 1$ 

#### Limitations of pLSA

- Not a proper generative model
  - $\boldsymbol{\theta}_d$  is treated as a parameter
  - Cannot model new documents

#### • Solution:

• Make it a proper generative model by adding priors to  $\theta$  and  $\beta$ 

### Limitations of pLSA

- Not a proper generative model
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• Solution:

• Make it a proper generative model by adding priors to  $\theta$  and  $\beta$ 

Topic Model v3: Latent Dirichlet Allocation (LDA)

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#### **Review:** Dirichlet Distribution

• Dirichlet distribution:  $\theta \sim Dirichlet(\alpha)$ 

• *i.e.*, 
$$p(\theta|\alpha) = \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} \prod_k \theta_k^{\alpha_k - 1}$$
, where  $\alpha_k > 0$   
•  $\Gamma(\cdot)$  is gamma function:  $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$   
•  $\Gamma(z+1) = z\Gamma(z)$ 



#### **Review: Dirichlet Distribution**

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•  $\Gamma(z+1) = z\Gamma(z)$ 

Simplex view:



36

34

#### More Examples in the Simplex View



# Topic Model v3: Latent Dirichlet Allocation (LDA)



 $\theta_d \sim Dirichlet(\alpha)$ : address topic distribution for unseen documents  $\beta_k \sim Dirichlet(\eta)$ : smoothing over words

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#### **Generative Model for LDA**

For each topic  $k \in \{1, \dots, K\}$ :  $\beta_k \sim \text{Dir}(\eta)$  [draw distribution over words] For each document  $d \in \{1, \dots, D\}$   $\theta_d \sim \text{Dir}(\alpha)$  [draw distribution over topics] For each word  $n \in \{1, \dots, N_d\}$   $z_{d,n} \sim \text{Mult}(1, \theta_d)$  [draw topic assignment]  $w_{d,n} \sim \theta_{z_{d,n}}$  [draw word]





- The generative story begins with only a Dirichlet prior over the topics.
- Each topic is defined as a Multinomial distribution over the vocabulary, parameterized by β<sub>k</sub>



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## LDA for Topic Modeling



 A topic is visualized as its high probability words.



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#### Joint Distribution for LDA



• Joint distribution of latent variables and documents is:  $p(\boldsymbol{\beta}_{1:K}, \boldsymbol{z}_{1:D}, \boldsymbol{\theta}_{1:D}, \boldsymbol{w}_{1:D} | \alpha, \eta) =$  $\prod_{i=1}^{K} p(\beta_i | \eta) \prod_{d=1}^{D} p(\theta_d | \alpha) \left( \prod_{n=1}^{N} p(z_{d,n} | \theta_d) p(w_{d,n} | \beta_{1:K}, z_{d,n}) \right)$ 

## **Questions?**