

DSC190: Machine Learning with Few Labels

Reinforcement learning

Zhiting Hu

Lecture 13, November 4, 2021

UC San Diego

HALICIOĞLU DATA SCIENCE INSTITUTE

Recap: Knowledge driven learning

- Two general ways of integrating structured knowledge with ML:
 - Model architecture (inductive bias)
 - Integrating knowledge through learning (loss, constraints)
 - Weak supervision
 - Posterior regularization
 - Integer linear programming (ILP)

Outline

- Reinforcement Learning
- 1 Paper presentations (15 mins)
 - **Kewen Zhao:** Task-Agnostic Meta-Learning for Few-shot Learning

So far... Supervised Learning

Data: (x, y)

x is data, y is label

Goal: Learn a *function* to map $x \rightarrow y$

Examples: Classification,
regression, object detection,
semantic segmentation, image
captioning, etc.



→ Cat

Classification

So far... Unsupervised Learning

Data: x
no labels!

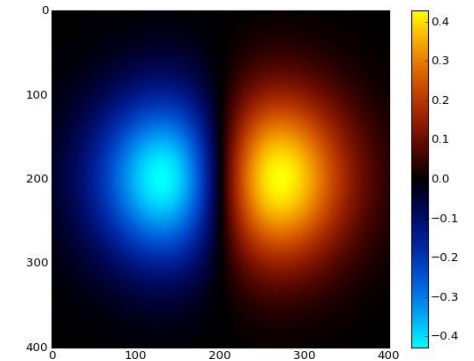
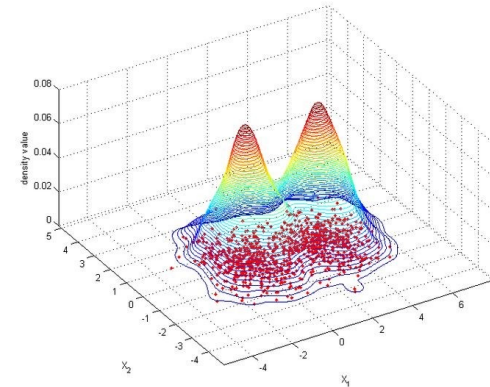
Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.



Figure copyright Ian Goodfellow, 2016. Reproduced with permission.

1-d density estimation

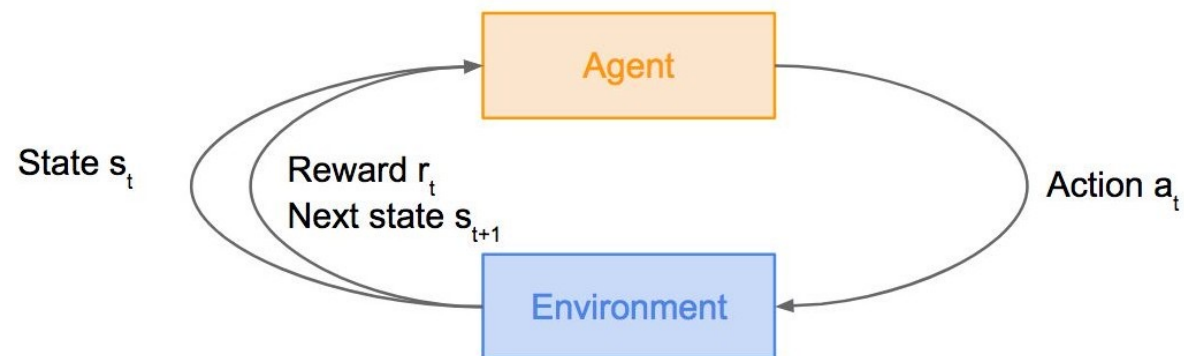


2-d density estimation

Today: Reinforcement Learning

Problems involving an **agent** interacting with an **environment**, which provides numeric **reward** signals

Goal: Learn how to take actions in order to maximize reward



Overview

- What is Reinforcement Learning?
- Markov Decision Processes
- Q-Learning
- Policy Gradients

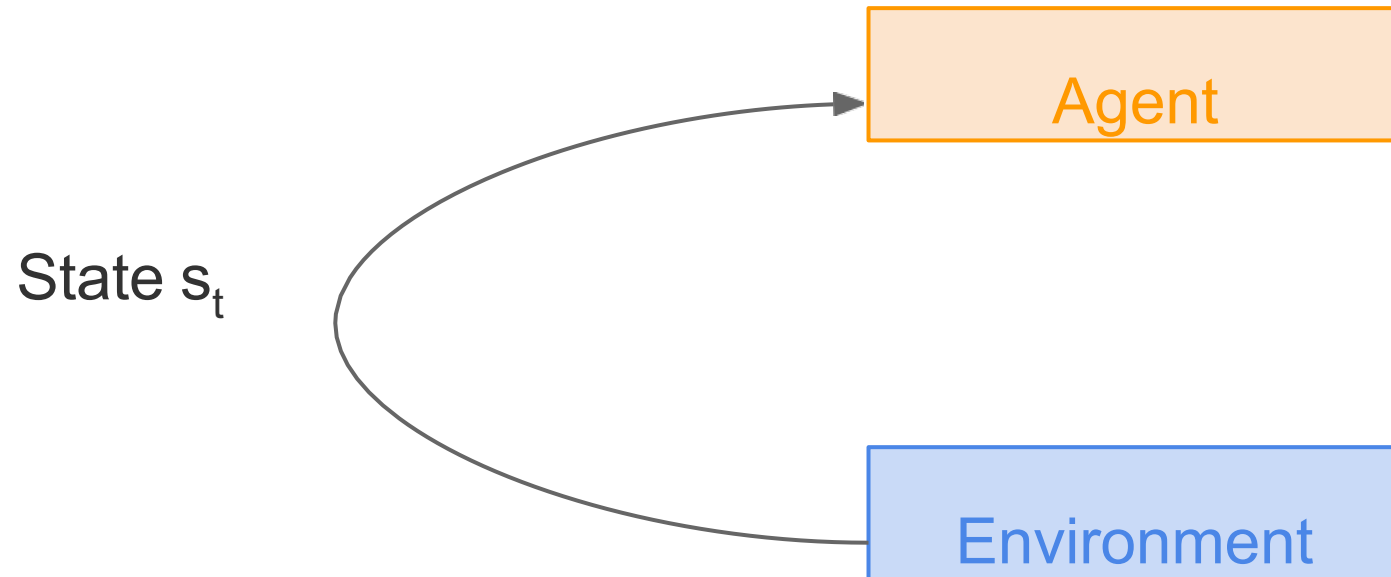
Reinforcement Learning

The diagram illustrates the core components of Reinforcement Learning. It features two rectangular boxes stacked vertically. The top box is light orange with an orange border and contains the word 'Agent' in orange text. The bottom box is light blue with a blue border and contains the word 'Environment' in blue text. This visualizes the interaction between the learning agent and the environment it operates in.

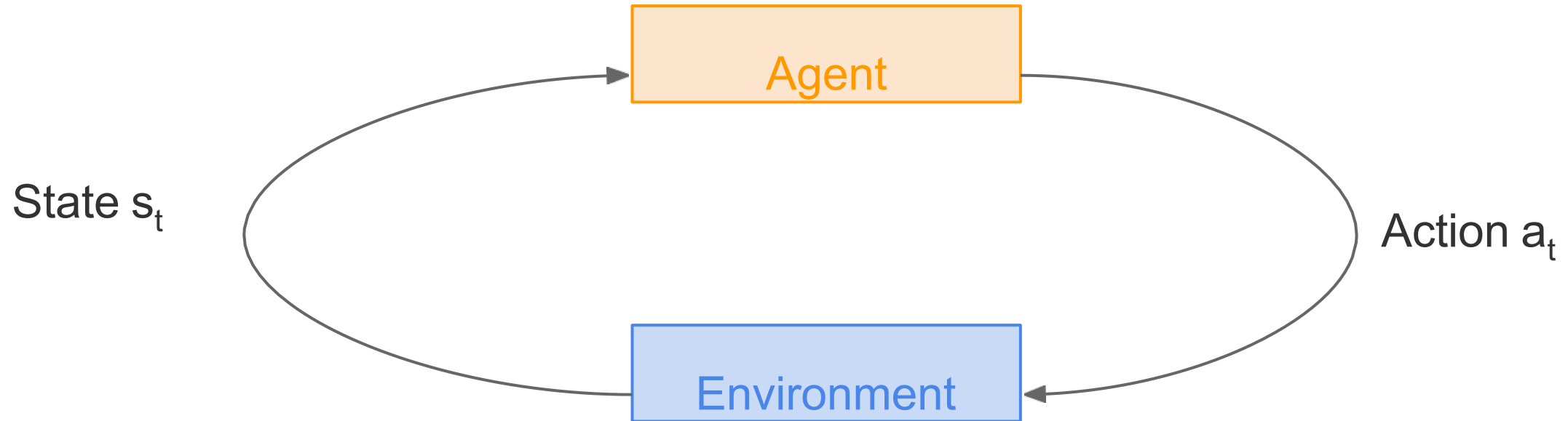
Agent

Environment

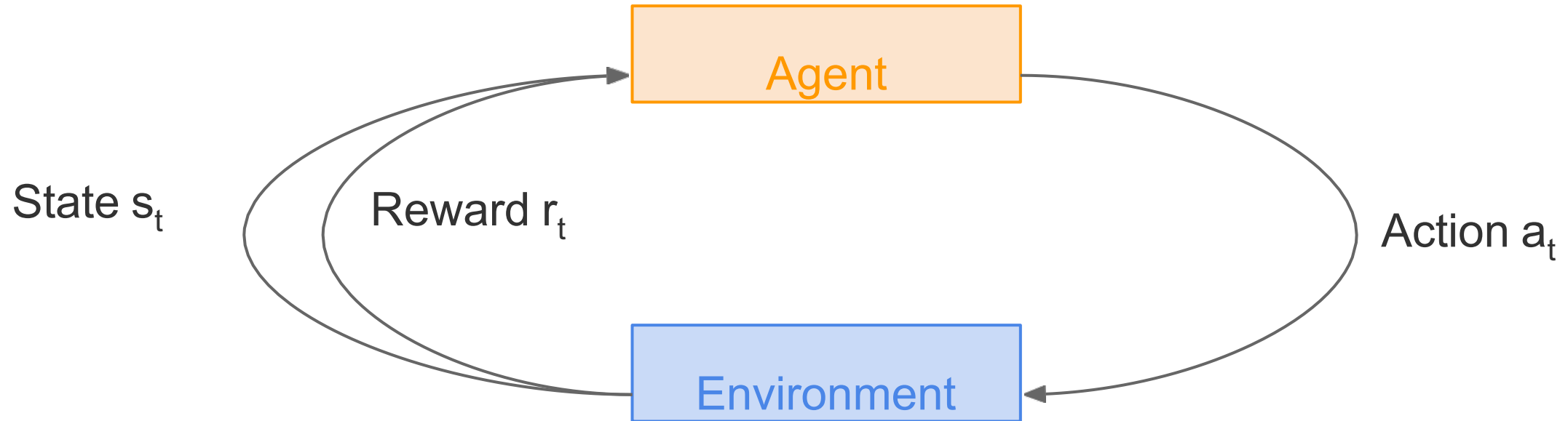
Reinforcement Learning



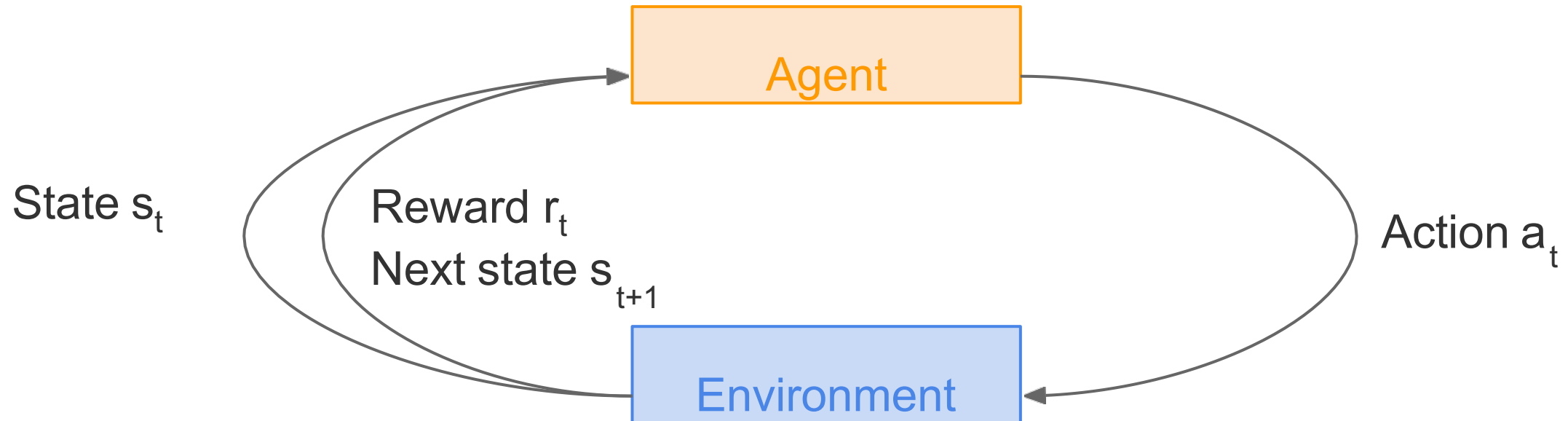
Reinforcement Learning



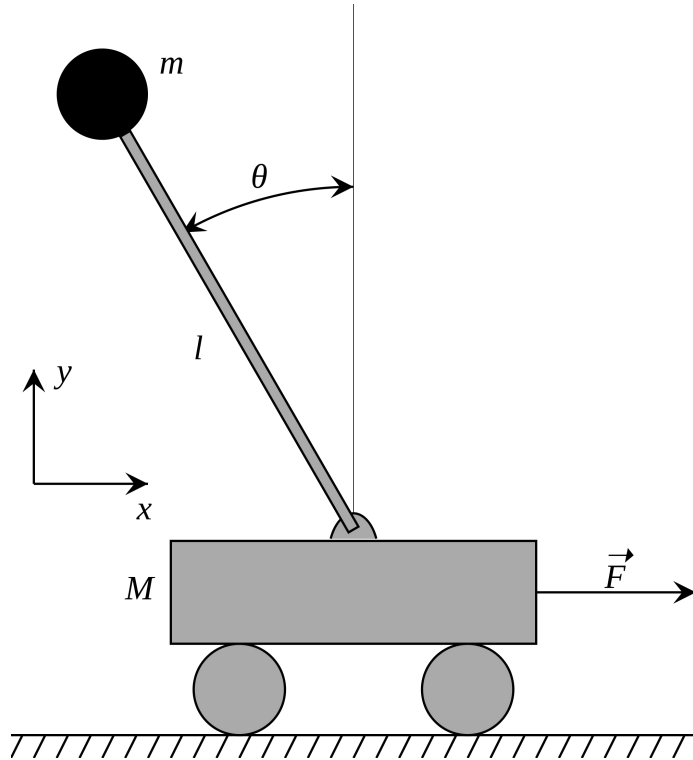
Reinforcement Learning



Reinforcement Learning



Cart-Pole Problem



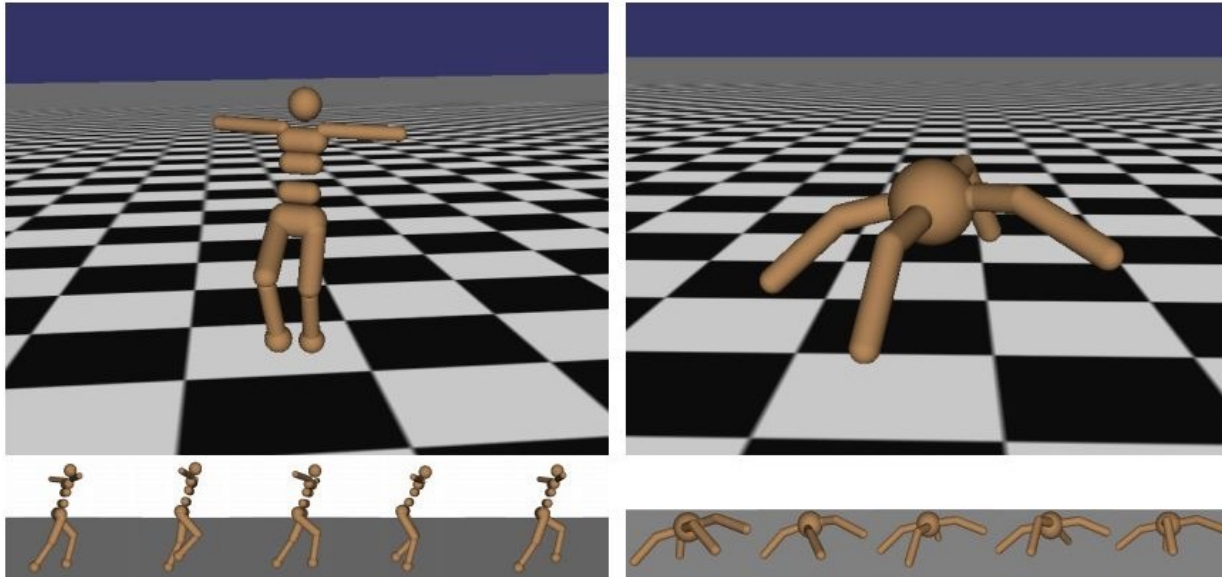
Objective: Balance a pole on top of a movable cart

State: angle, angular speed, position, horizontal velocity

Action: horizontal force applied on the cart

Reward: 1 at each time step if the pole is upright

Robot Locomotion



Objective: Make the robot move forward

State: Angle and position of the joints

Action: Torques applied on joints

Reward: 1 at each time step upright + forward movement

Atari Games



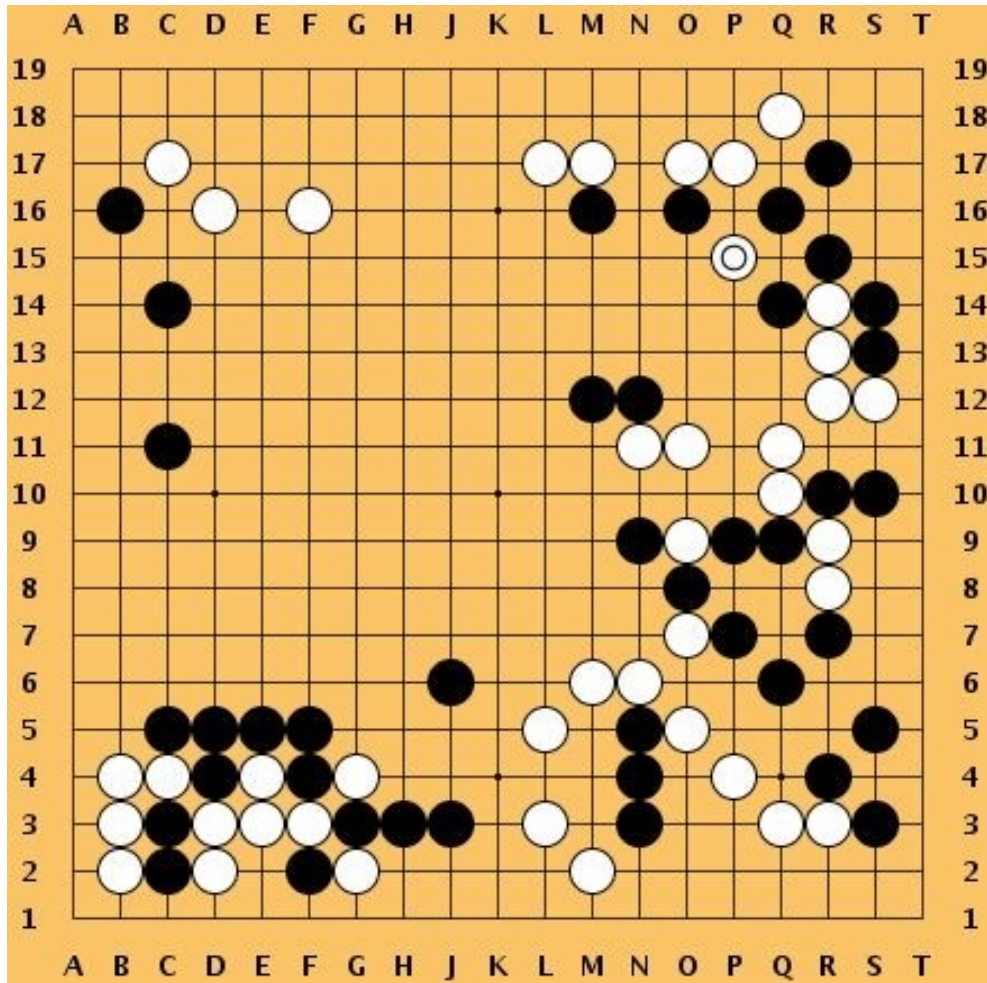
Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

Go



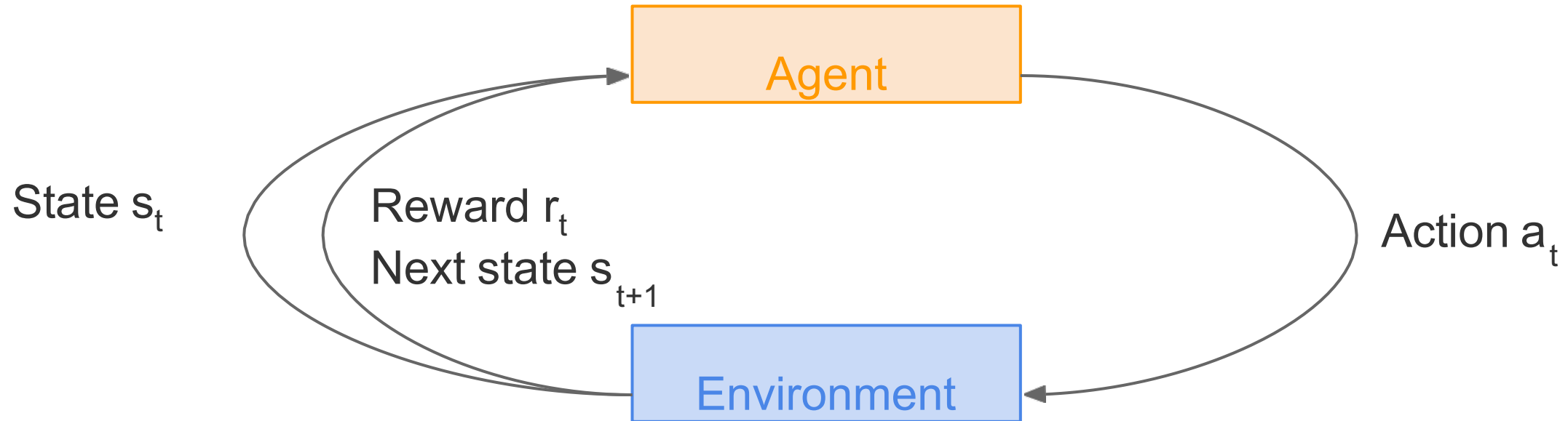
Objective: Win the game!

State: Position of all pieces

Action: Where to put the next piece down

Reward: 1 if win at the end of the game, 0 otherwise

How can we mathematically formalize the RL problem?



Markov Decision Process

- Mathematical formulation of the RL problem
- **Markov property**: Current state completely characterises the state of the world

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

\mathcal{S} : set of possible states

\mathcal{A} : set of possible actions

\mathcal{R} : distribution of reward given (state, action) pair

\mathbb{P} : transition probability i.e. distribution over next state given (state, action) pair

γ : discount factor

Markov Decision Process

- At time step $t=0$, environment samples initial state $s_0 \sim p(s_0)$
- Then, for $t=0$ until done:
 - Agent selects action a_t
 - Environment samples reward $r_t \sim R(\cdot | s_t, a_t)$
 - Environment samples next state $s_{t+1} \sim P(\cdot | s_t, a_t)$
 - Agent receives reward r_t and next state s_{t+1}
- A policy π is a function from S to A that specifies what action to take in each state
- **Objective:** find policy π^* that maximizes cumulative discounted reward: $\sum_{t \geq 0} \gamma^t r_t$

A simple MDP: Grid World

actions = {

1. right 

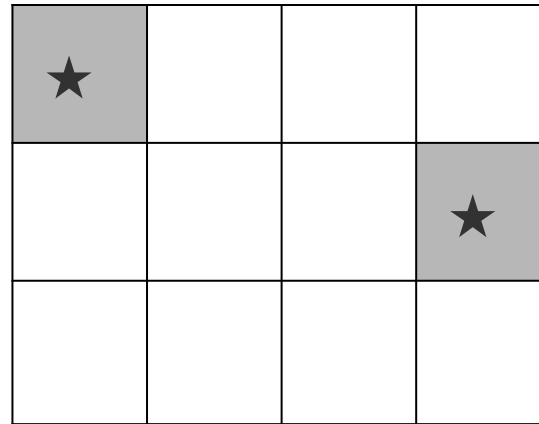
2. left 

3. up 

4. down 

}

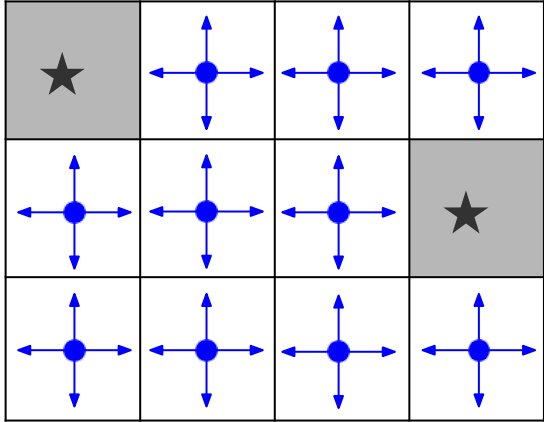
states



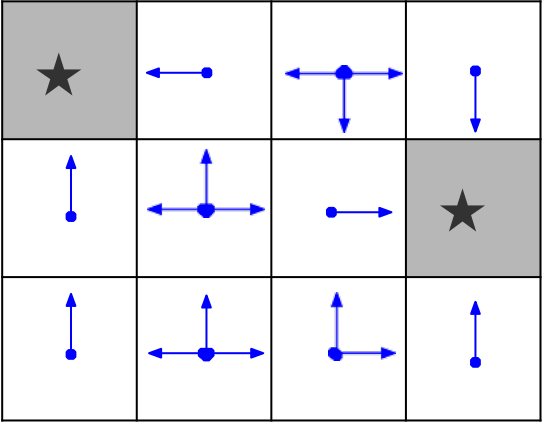
Set a negative “reward”
for each transition
(e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in
least number of actions

A simple MDP: Grid World



Random Policy



Optimal Policy

The optimal policy π^*

We want to find optimal policy π^* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)?

The optimal policy π^*

We want to find optimal policy π^* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)?

Maximize the **expected sum of rewards!**

$$\text{Formally: } \pi^* = \arg \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | \pi \right] \text{ with } s_0 \sim p(s_0), a_t \sim \pi(\cdot | s_t), s_{t+1} \sim p(\cdot | s_t, a_t)$$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

How good is a state?

The **value function** at state s , is the expected cumulative reward from following the policy from state s :

$$V^\pi(s) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, \pi \right]$$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

How good is a state?

The **value function** at state s , is the expected cumulative reward from following the policy from state s :

$$V^\pi(s) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, \pi \right]$$

How good is a state-action pair?

The **Q-value function** at state s and action a , is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s, a) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Q^* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') \mid s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s', a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Q^* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') \mid s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s', a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

The optimal policy π^* corresponds to taking the best action in any state as specified by Q^*

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') \mid s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \infty$

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') \mid s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \infty$

What's the problem with this?

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') \mid s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \infty$

What's the problem with this?

Not scalable. Must compute $Q(s,a)$ for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') | s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \infty$

What's the problem with this?

Not scalable. Must compute $Q(s,a)$ for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solution: use a function approximator to estimate $Q(s,a)$. E.g. a neural network!

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

If the function approximator is a deep neural network => **deep q-learning!**

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning!**

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} \left[(y_i - Q(s, a; \theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} \left[(y_i - Q(s, a; \theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} \left[(y_i - Q(s, a; \theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

close to the target value (y) it should have, if Q-function corresponds to optimal Q* (and optimal policy π^*)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$

Case Study: Playing Atari Games



Objective: Complete the game with the highest score

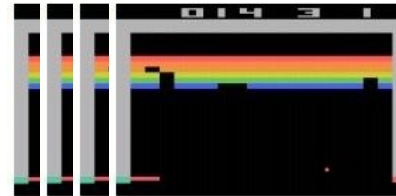
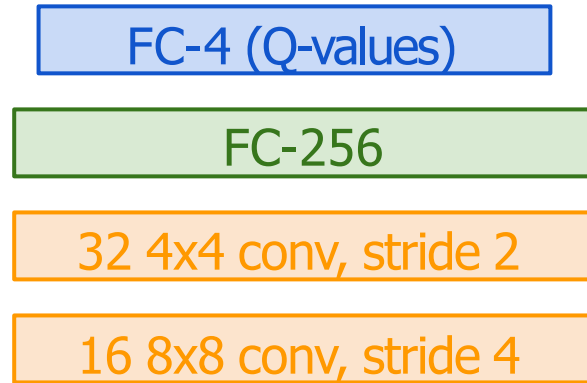
State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

Q-network Architecture

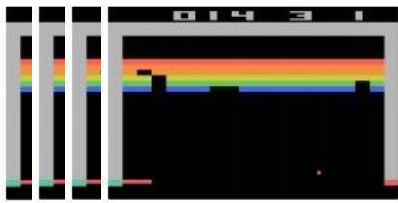
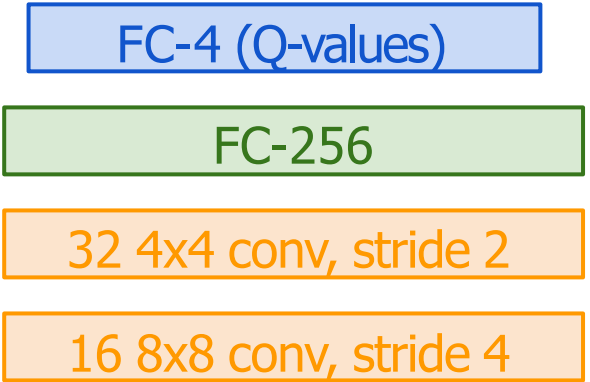
$Q(s, a; \theta)$:
neural network
with weights θ



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ

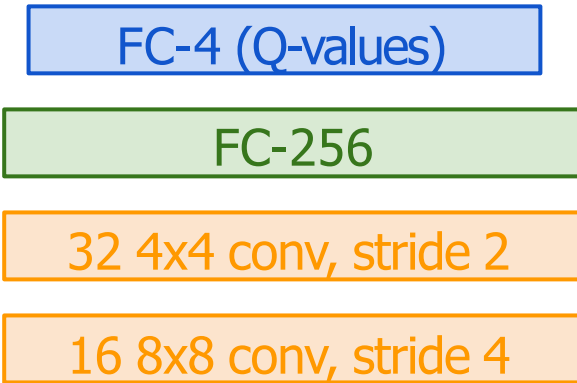


← Input: state s_t

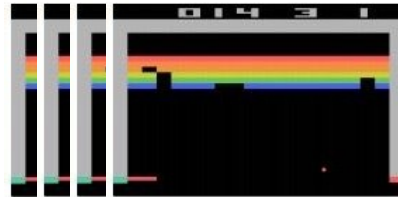
Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



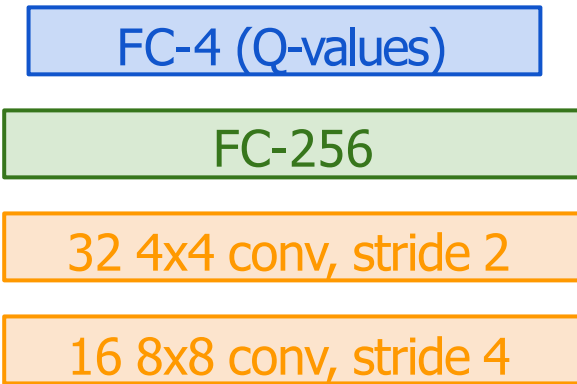
← Familiar conv layers,
FC layer



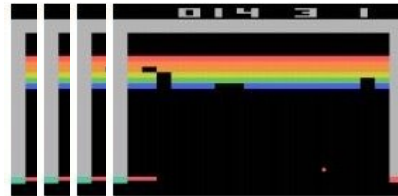
Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



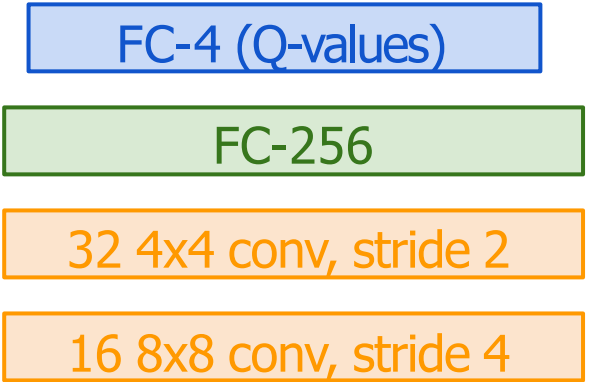
← Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



← Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$

Number of actions between 4-18 depending on Atari game

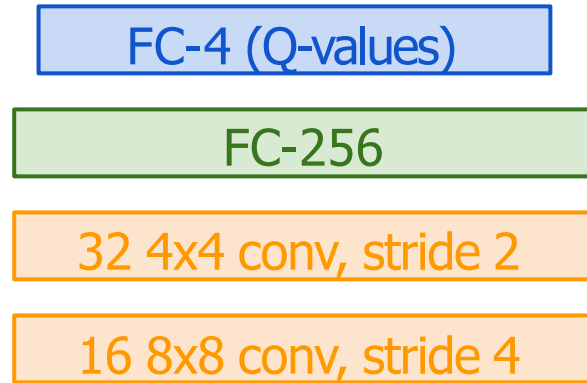


Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

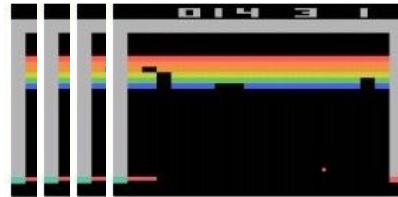
$Q(s, a; \theta)$:
neural network
with weights θ

A single feedforward pass
to compute Q-values for all
actions from the current
state => efficient!



← Last FC layer has 4-d
output (if 4 actions),
corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$,
 $Q(s_t, a_4)$

Number of actions between 4-18
depending on Atari game



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Recap: Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} \left[(y_i - Q(s, a; \theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

close to the target value (y) it should have, if Q-function corresponds to optimal Q* (and optimal policy π^*)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

Each transition can also contribute to multiple weight updates
=> greater data efficiency

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

← Initialize replay memory, Q-network

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

← Play M episodes (full games)

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ ←

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Initialize state
(starting game
screen pixels) at the
beginning of each
episode

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for



For each timestep t
of the game

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

← With small probability, select a random action (explore), otherwise select greedy action from current policy

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

← Take the action (a_t), and observe the reward r_t and next state s_{t+1}

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for



Store transition in
replay memory

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

← Experience Replay:
Sample a random minibatch of transitions from replay memory and perform a gradient descent step

Policy Gradients

What is a problem with Q-learning?

The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

Policy Gradients

What is a problem with Q-learning?

The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand

Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta \right]$$

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta \right]$$

We want to find the optimal policy $\theta^* = \arg \max_{\theta} J(\theta)$

How can we do this?

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta \right]$$

We want to find the optimal policy $\theta^* = \arg \max_{\theta} J(\theta)$

How can we do this?

Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$\begin{aligned} J(\theta) &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)] \\ &= \int_{\tau} r(\tau) p(\tau; \theta) d\tau \end{aligned}$$

Where $r(r)$ is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \dots)$

REINFORCE algorithm

Expected reward:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)]$$
$$= \int_{\tau} r(\tau) p(\tau; \theta) d\tau$$

REINFORCE algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)]$

$$= \int_{\tau} r(\tau) p(\tau; \theta) d\tau$$

Now let's differentiate this: $\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$

REINFORCE algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)]$

$$= \int_{\tau} r(\tau) p(\tau; \theta) d\tau$$

Now let's differentiate this: $\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$

Intractable! Gradient of an expectation is problematic when p depends on θ

REINFORCE algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)]$

$$= \int_{\tau} r(\tau) p(\tau; \theta) d\tau$$

Now let's differentiate this: $\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$

Intractable! Gradient of an expectation is problematic when p depends on θ

However, we can use a nice trick: $\nabla_{\theta} p(\tau; \theta) = p(\tau; \theta) \frac{\nabla_{\theta} p(\tau; \theta)}{p(\tau; \theta)} = p(\tau; \theta) \nabla_{\theta} \log p(\tau; \theta)$

REINFORCE algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)]$

$$= \int_{\tau} r(\tau) p(\tau; \theta) d\tau$$

Now let's differentiate this: $\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$

Intractable! Gradient of an expectation is problematic when p depends on θ

However, we can use a nice trick: $\nabla_{\theta} p(\tau; \theta) = p(\tau; \theta) \frac{\nabla_{\theta} p(\tau; \theta)}{p(\tau; \theta)} = p(\tau; \theta) \nabla_{\theta} \log p(\tau; \theta)$

If we inject this back:

$$\begin{aligned} \nabla_{\theta} J(\theta) &= \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau \\ &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)] \end{aligned}$$

Can estimate with Monte Carlo sampling

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$

Thus: $\log p(\tau; \theta) = \sum_{t \geq 0} \log p(s_{t+1} | s_t, a_t) + \log \pi_{\theta}(a_t | s_t)$

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$

Thus: $\log p(\tau; \theta) = \sum_{t \geq 0} \log p(s_{t+1} | s_t, a_t) + \log \pi_{\theta}(a_t | s_t)$

And when differentiating: $\nabla_{\theta} \log p(\tau; \theta) = \sum_{t \geq 0} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

Doesn't depend on
transition probabilities!

REINFORCE algorithm

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau \\ &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)]\end{aligned}$$

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$

Thus: $\log p(\tau; \theta) = \sum_{t \geq 0} \log p(s_{t+1} | s_t, a_t) + \log \pi_{\theta}(a_t | s_t)$

And when differentiating: $\nabla_{\theta} \log p(\tau; \theta) = \sum_{t \geq 0} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

Doesn't depend on transition probabilities!

Therefore when sampling a trajectory r , we can estimate $J(\theta)$ with

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Intuition

Gradient estimator: $\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

Interpretation:

- If $r(r)$ is high, push up the probabilities of the actions seen
- If $r(r)$ is low, push down the probabilities of the actions seen

Intuition

Gradient estimator:
$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Interpretation:

- If $r(r)$ is high, push up the probabilities of the actions seen
- If $r(r)$ is low, push down the probabilities of the actions seen

Might seem simplistic to say that if a trajectory is good then all its actions were good. **But in expectation, it averages out!**

Intuition

Gradient estimator: $\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

Interpretation:

- If $r(r)$ is high, push up the probabilities of the actions seen
- If $r(r)$ is low, push down the probabilities of the actions seen

Might seem simplistic to say that if a trajectory is good then all its actions were good. **But in expectation, it averages out!**

However, this also suffers from high variance because credit assignment is really hard. Can we help the estimator?

Variance reduction

Gradient estimator: $\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

Variance reduction

Gradient estimator:
$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Variance reduction

Gradient estimator:
$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Second idea: Use discount factor γ to ignore delayed effects

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Variance reduction: Baseline

Problem: The raw value of a trajectory isn't necessarily meaningful. For example, if rewards are all positive, you keep pushing up probabilities of actions.

What is important then? Whether a reward is better or worse than what you expect to get

Idea: Introduce a baseline function dependent on the state.
Concretely, estimator is now:

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

How to choose the baseline?

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

A simple baseline: constant moving average of rewards experienced so far from all trajectories

How to choose the baseline?

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

A simple baseline: constant moving average of rewards experienced so far from all trajectories

Variance reduction techniques seen so far are typically used in “Vanilla REINFORCE”

How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

A: Q-function and value function!

How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

A: Q-function and value function!

Intuitively, we are happy with an action a_t in a state s_t if $Q^\pi(s_t, a_t) - V^\pi(s_t)$ is large. On the contrary, we are unhappy with an action if it's small.

How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

A: Q-function and value function!

Intuitively, we are happy with an action a_t in a state s_t if $Q^\pi(s_t, a_t) - V^\pi(s_t)$ is large. On the contrary, we are unhappy with an action if it's small.

Using this, we get the estimator:
$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} (Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)) \nabla_\theta \log \pi_\theta(a_t | s_t)$$

Actor-Critic Algorithm

Problem: we don't know Q and V. Can we learn them?

Yes, using Q-learning! We can combine Policy Gradients and Q-learning by training both an **actor** (the policy) and a **critic** (the Q-function).

- The actor decides which action to take, and the critic tells the actor how good its action was and how it should adjust
- Also alleviates the task of the critic as it only has to learn the values of (state, action) pairs generated by the policy
- Can also incorporate Q-learning tricks e.g. experience replay
- **Remark:** we can define by the **advantage function** how much an action was better than expected

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$$

Actor-Critic Algorithm

Initialize policy parameters θ , critic parameters ϕ

For iteration=1, 2 ... **do**

 Sample m trajectories under the current policy

$\Delta\theta \leftarrow 0$

For $i=1, \dots, m$ **do**

For $t=1, \dots, T$ **do**

$$A_t = \sum_{t' \geq t} \gamma^{t'-t} r_{t'}^i - V_{\phi}(s_t^i)$$

$$\Delta\theta \leftarrow \Delta\theta + A_t \nabla_{\theta} \log(a_t^i | s_t^i)$$

$$\Delta\phi \leftarrow \sum_i \sum_t \nabla_{\phi} \|A_t^i\|^2$$

$$\theta \leftarrow \alpha \Delta\theta$$

$$\phi \leftarrow \beta \Delta\phi$$

End for

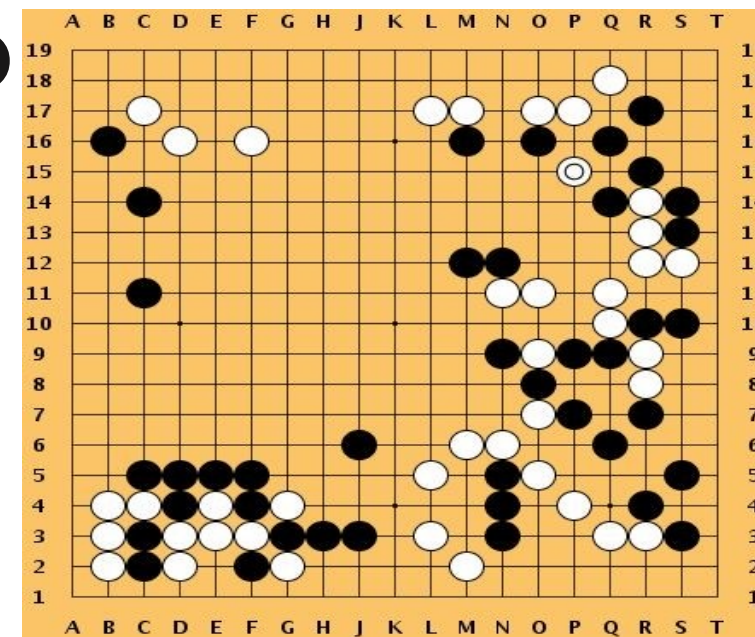
More policy gradients: AlphaGo

Overview:

- Mix of supervised learning and reinforcement learning
- Mix of old methods (Monte Carlo Tree Search) and recent ones (deep RL)

How to beat the Go world champion:

- Featurize the board (stone color, move legality, bias, ...)
- Initialize policy network with supervised training from professional go games, then continue training using policy gradient (play against itself from random previous iterations, +1 / -1 reward for winning / losing)
- Also learn value network (critic)
- Finally, combine combine policy and value networks in a Monte Carlo Tree Search algorithm to select actions by lookahead search



*[Silver et al.,
Nature 2016]*

This image is [CC0 public domain](#)

Key Takeaways

- Markov Decision Process (MDP)
- Q-learning
 - Bellman equation
 - Deep Q-learning, experience replay
- Policy gradients
- Guarantees:
 - Policy Gradients: Converges to a local minima of $J(\theta)$, often good enough!
 - Q-learning: Zero guarantees since you are approximating Bellman equation with a complicated function approximator

Questions?